

Bits of Language: Architecture

Overview

- **Last two weeks.** Deep learning for **images**
 - Architecture. ConvNet
 - Training. Augmentation & Self-supervised training
 - Generation. VAE, GAN, Diffusion
- **This week.** Deep learning for **text**
 - Architecture. Preprocessing, RNN, Transformer
 - Generation. BERT & GPT

Overview

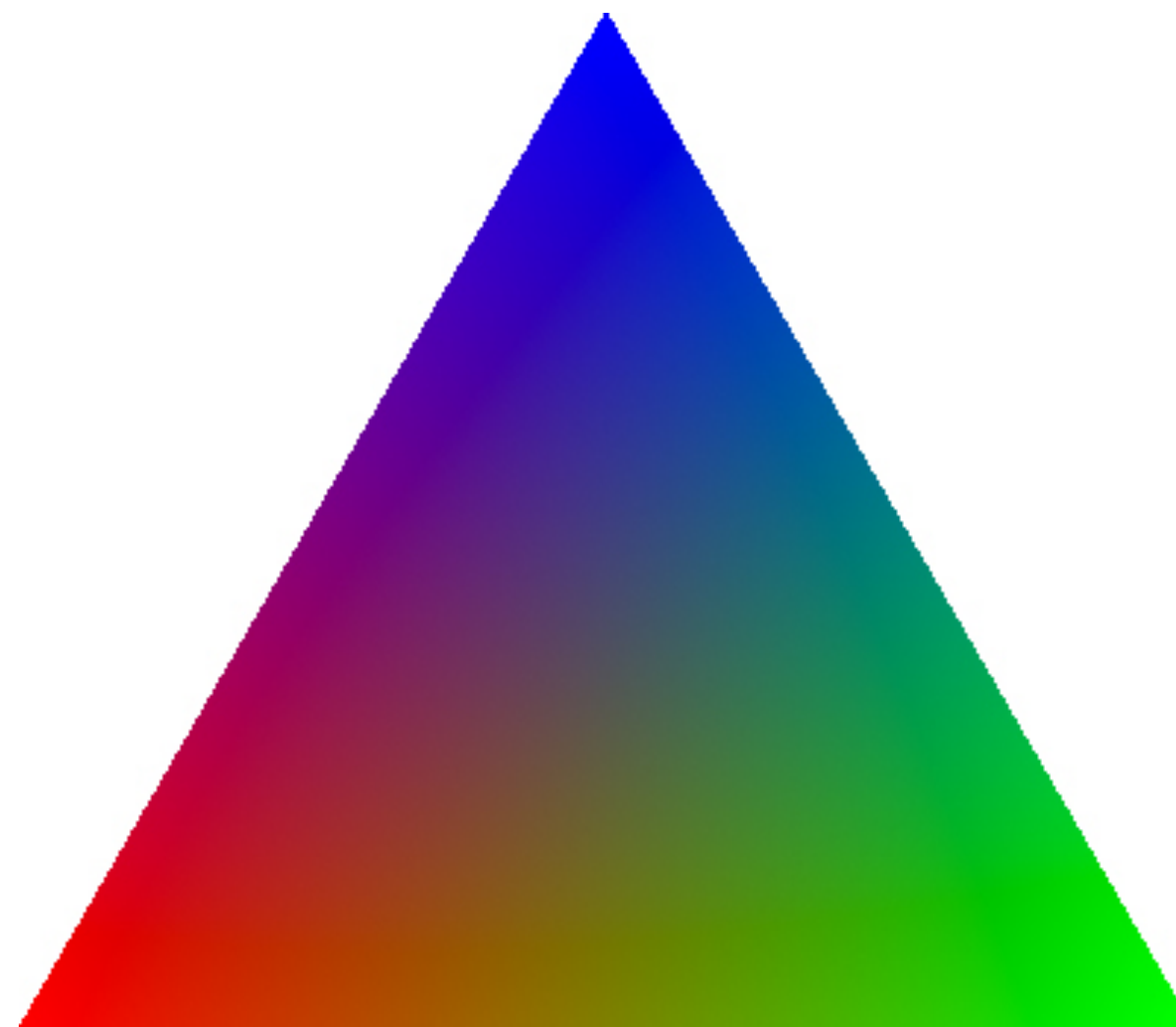
- **Last two weeks.** Deep learning for **images**
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Preview: Text vs. Image

- Text and image differs in many aspects

(1) Text is **discrete**

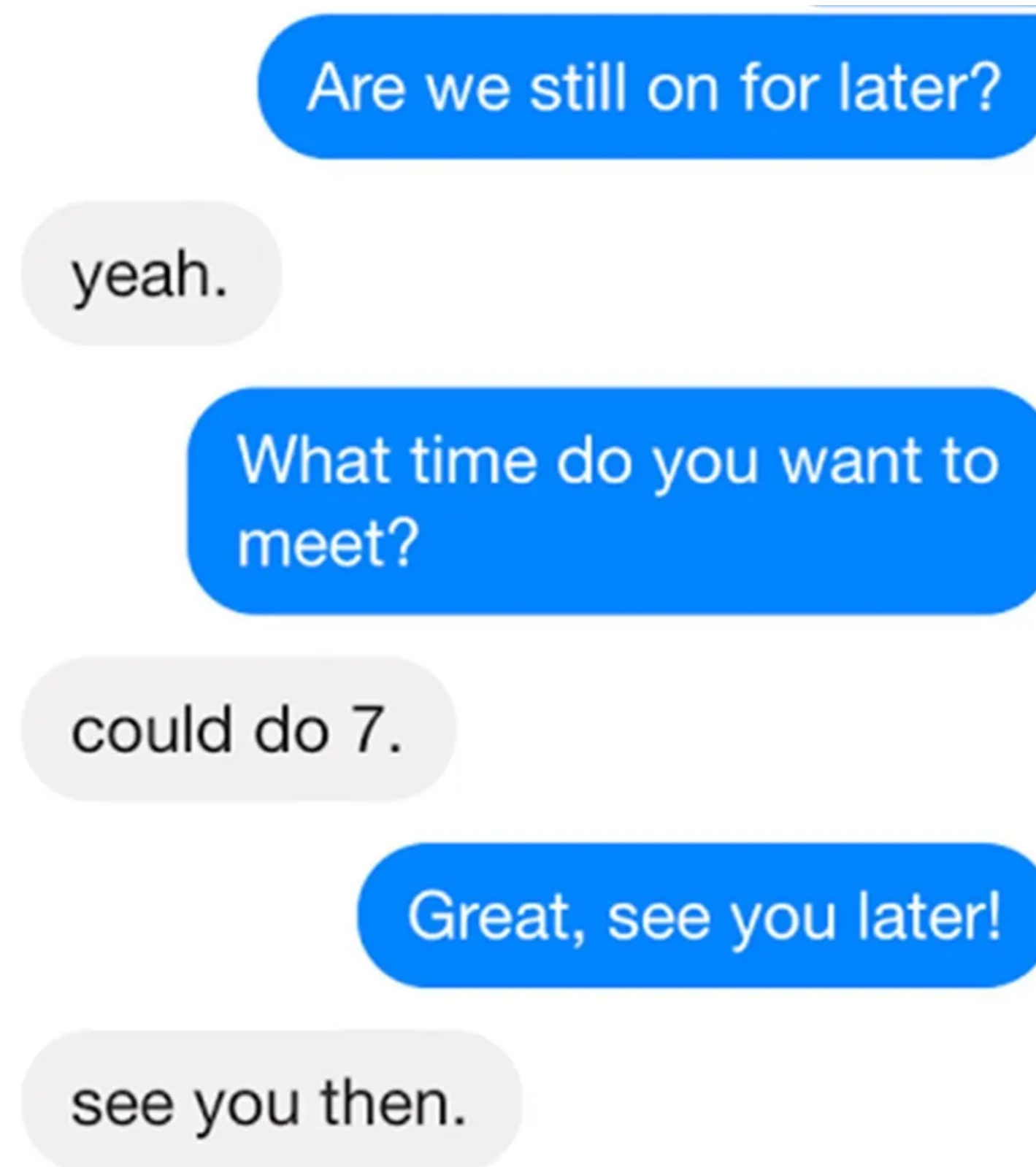
- Interpolating “■” & “■” vs. “A” & “C”
- Required. A nice text vectorization mechanism



Preview: Text vs. Image

(2) Text has **variable length**

- $\Leftrightarrow$ image with fixed resolution – or can do downsampling
- Required. An architecture that can handle sequences effectively



Preview: Text vs. Image

(3) Text has **weaker locality** than images

- ⇔ image with high locality
- Required. Architecture that can cover far distances

“**The boy** did not have
any idea where **he** is at.”



Text preprocessing

Preprocessing

- Unlike images, **translating text into vectors** is not straightforward
 - Unicode? ASCII? A vs. B vs. C

- Typically goes through:

- Normalization
- Pre-tokenization
- Tokenization
- Embedding

“The boy did not have
any idea where he is at.”



$(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n), \quad \mathbf{x}_i \in \mathbb{R}^d$



Model

Preprocessing

- The first three steps (normalization ~ tokenization) are responsible for chunking the text and mapping them into codes.

Tokens	Characters
31	137

There are plenty of different ways to tokenize the text into multiple pieces. GPT-4o and GPT-3.5 are actually using different tokenizers.

Text Token IDs

[5632, 553, 13509, 328, 2647, 6984, 316, 192720, 290, 2201, 1511, 7598, 12762, 13, 174803, 12, 19, 78, 326, 174803, 12, 18, 13, 20, 553, 4771, 2360, 2647, 6602, 24223, 13]

Text Token IDs

Preprocessing

- The last step (embedding) maps each chunk to a vector
 - Want to keep our dictionary small enough for handling

```
[5632, 553, 13509, 328, 2647, 6984, 316, 192720, 290, 2201, 1511, 7598,  
12762, 13, 174803, 12, 19, 78, 326, 174803, 12, 18, 13, 20, 553, 4771,  
2360, 2647, 6602, 24223, 13]
```

Text

Token IDs

[token 1] $\longrightarrow \mathbf{x}_1 \in \mathbb{R}^d$

[token 2] $\longrightarrow \mathbf{x}_2 \in \mathbb{R}^d$

...

[token 30522] $\longrightarrow \mathbf{x}_{30522} \in \mathbb{R}^d$

Step 1. Normalization

- Various cleanups on the given text to reduce the data complexity
 - Remove unnecessary variations
 - Hello → hello # uppercase
 - I ate it all → I ate it all # whitespace
 - café → cafe # accent
 - e-mail → email # punctuation
 - Unify date & numeric formats
 - 01/31/2024 → 2024-01-31
 - 31st Jan. 2024 → 2024-01-31

Step 1. Normalization

- Often, this is done at a unicode level
 - There are many equivalences...
 - <https://www.unicode.org/reports/tr15/>
- **Note.** Some LLMs are known to use a specific unicode for “ ”
 - Easy to filter out LLM-generated data from their own training data
 - Copyright uses
 - Catching cheating ;)

Subtype	Examples		
Font variants	ℹ	→	H
	ℹ	→	H
Linebreaking differences	[NBSP] → [SPACE]		
Positional variant forms	ε	→	ε
	ε	→	ε
	ε	→	ε
	ε	→	ε
Circled variants	①	→	1
Width variants	カ	→	カ
Rotated variants	↵	→	{
	↵	→	}
Superscripts/subscripts	i ⁹	→	i ₉
	i ₉	→	i ⁹
Squared characters	アパート	→	アパート
Fractions	¼	→	1/4
Other	dž	→	dž

Step 2. Pre-tokenization

- **Break down** text into manageable units
 - Facilitate more accurate tokenization — i.e., chunking
 - Sometimes, prevent breaking down
- can't → can + 't # contraction
- some sentence. → some sentence + . # punctuation
- DMZ ↗ D + MZ # abbreviation & acronym

Step 3. Tokenization

- Break down each sentence into **tokens**
- Many variants...

(1) Word-based tokenization

- Good semantics
- Too many vocabularies

Split on spaces

Let's	do	tokenization!
-------	----	---------------

Split on punctuation

Let	's	do	tokenization	!
-----	----	----	--------------	---

Step 3. Tokenization

(2) Character-based tokenization

- Small vocabulary size
- Bad semantics

L	e	t	'	s	d	o	t	o	k	e	n	i	z	a	t	i	o	n	!
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Step 3. Tokenization

(3) Subword tokenization

- Frequent words are kept as a single token
- Rare words are subdivided
 - Reduces the expected sequence length
- How to take whitespaces into account differs from a tokenizer to another

Let's </w>

do</w>

token

ization</w>

!</w>

Step 3. Tokenization

- As an example, we'll take a look at **Byte-Pair Encoding** (BPE)
 - A data-driven method to generate subword tokenization policy
 - Similar: WordPiece
- **Idea.** Merge frequent character combinations into tokens
 - Begin from the character-level tokens
 - If certain token appears together frequently, merge them
 - Repeat

Tokenization: Byte-Pair Encoding

```
"hug", "pug", "pun", "bun", "hugs"
```

- **Example.** Suppose that our text corpus consists of five words

- Initial vocabulary: [b, g, h, n, p, s, u]
- Count the word frequencies: ("hug", 10), ("pug", 5), ("pun", 12), ("bun", 4), ("hugs", 5)
- Use this to count subword frequencies

```
("h" "u" "g", 10), ("p" "u" "g", 5), ("p" "u" "n", 12), ("b" "u" "n", 4), ("h" "u" "g" "s", 5)
```

Tokenization: Byte-Pair Encoding

```
"hug", "pug", "pun", "bun", "hugs"
```

- **Example.** Suppose that our text corpus consists of five words

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```
("h" "u" "g", 10), ("p" "u" "g", 5), ("p" "u" "n", 12), ("b" "u" "n", 4), ("h" "u" "g" "s", 5)
```

- Expand the vocabulary

```
Vocabulary: ["b", "g", "h", "n", "p", "s", "u", "ug"]
```

```
Corpus: ("h" "ug", 10), ("p" "ug", 5), ("p" "u" "n", 12), ("b" "u" "n", 4), ("h" "ug" "s", 5)
```

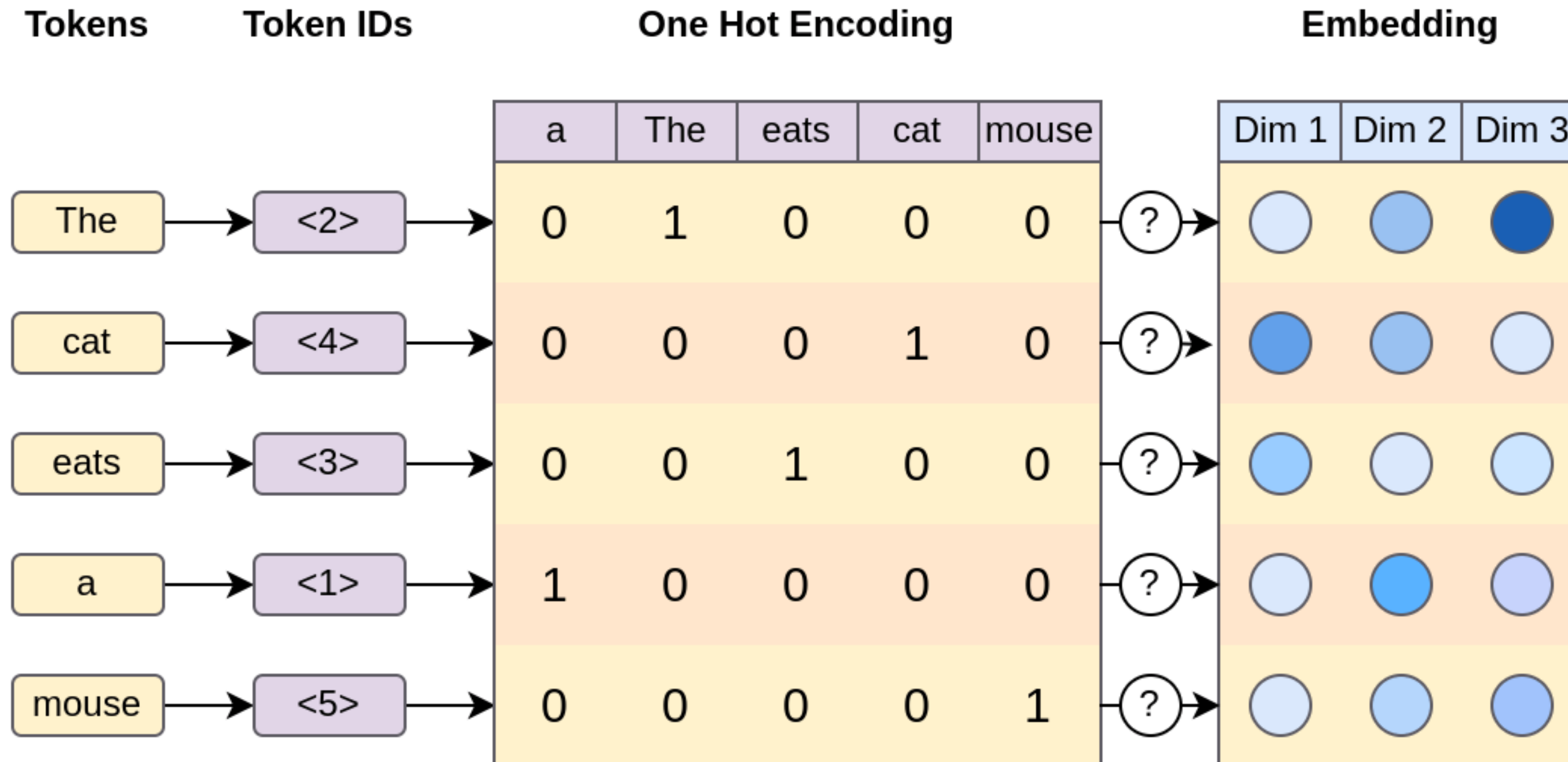
- Repeat, until the desired vocabulary size is met

```
Vocabulary: ["b", "g", "h", "n", "p", "s", "u", "ug", "un", "hug"]
```

```
Corpus: ("hug", 10), ("p" "ug", 5), ("p" "un", 12), ("b" "un", 4), ("hug" "s", 5)
```

Step 4. Embedding

- Token IDs are translated into one-hot encodings, and then to embeddings
 - Implementable with lookup tables
 - Embedding is **trainable** as well – more on this later

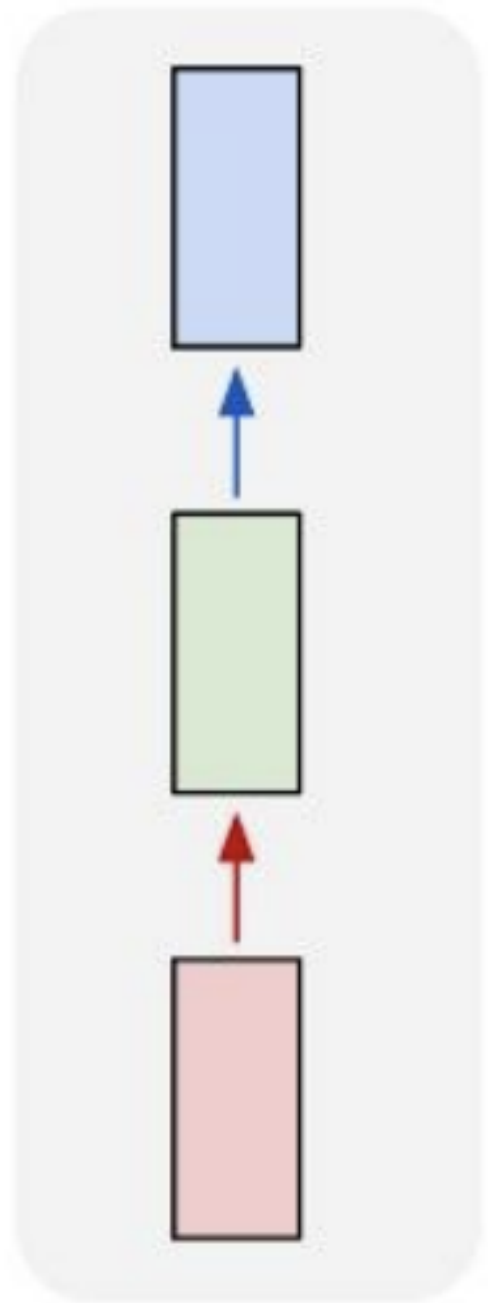


Architectures

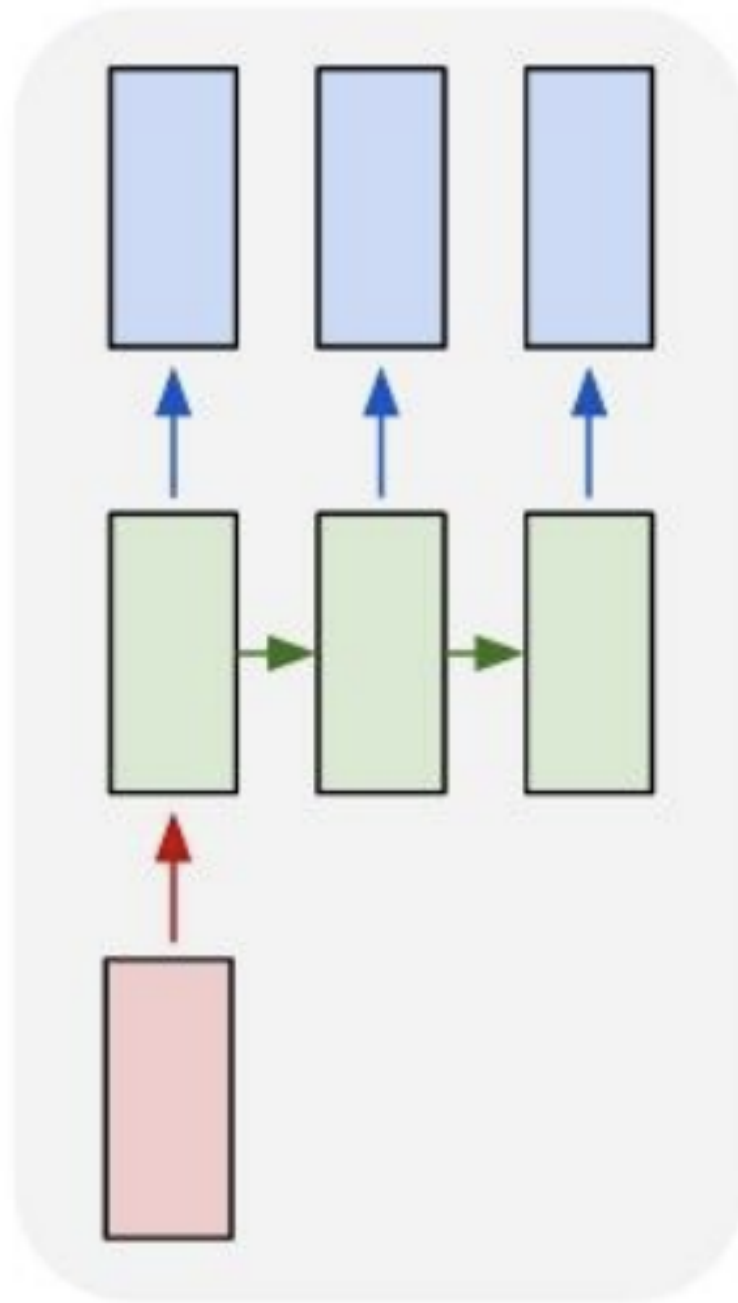
Architectures

- We will cover two architectures that are designed for **sequences**
 - Recurrent neural nets (RNNs)
 - Transformers

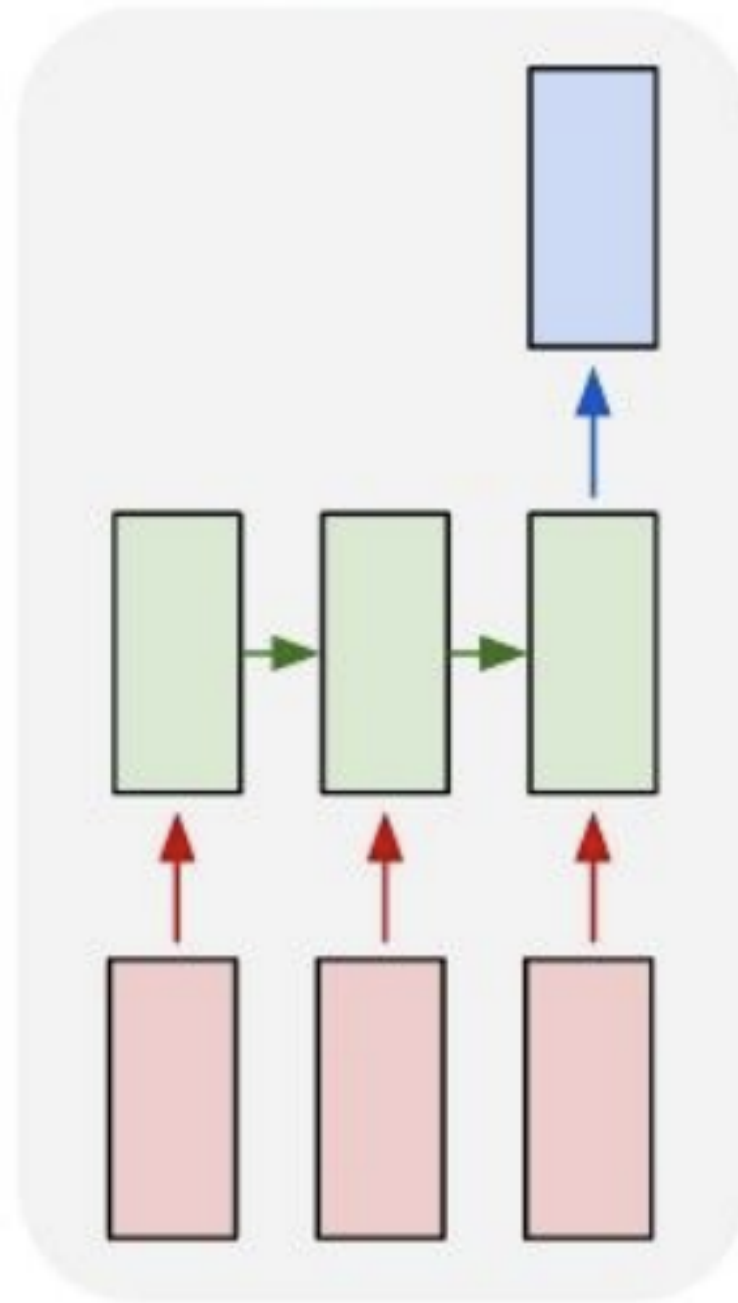
one to one



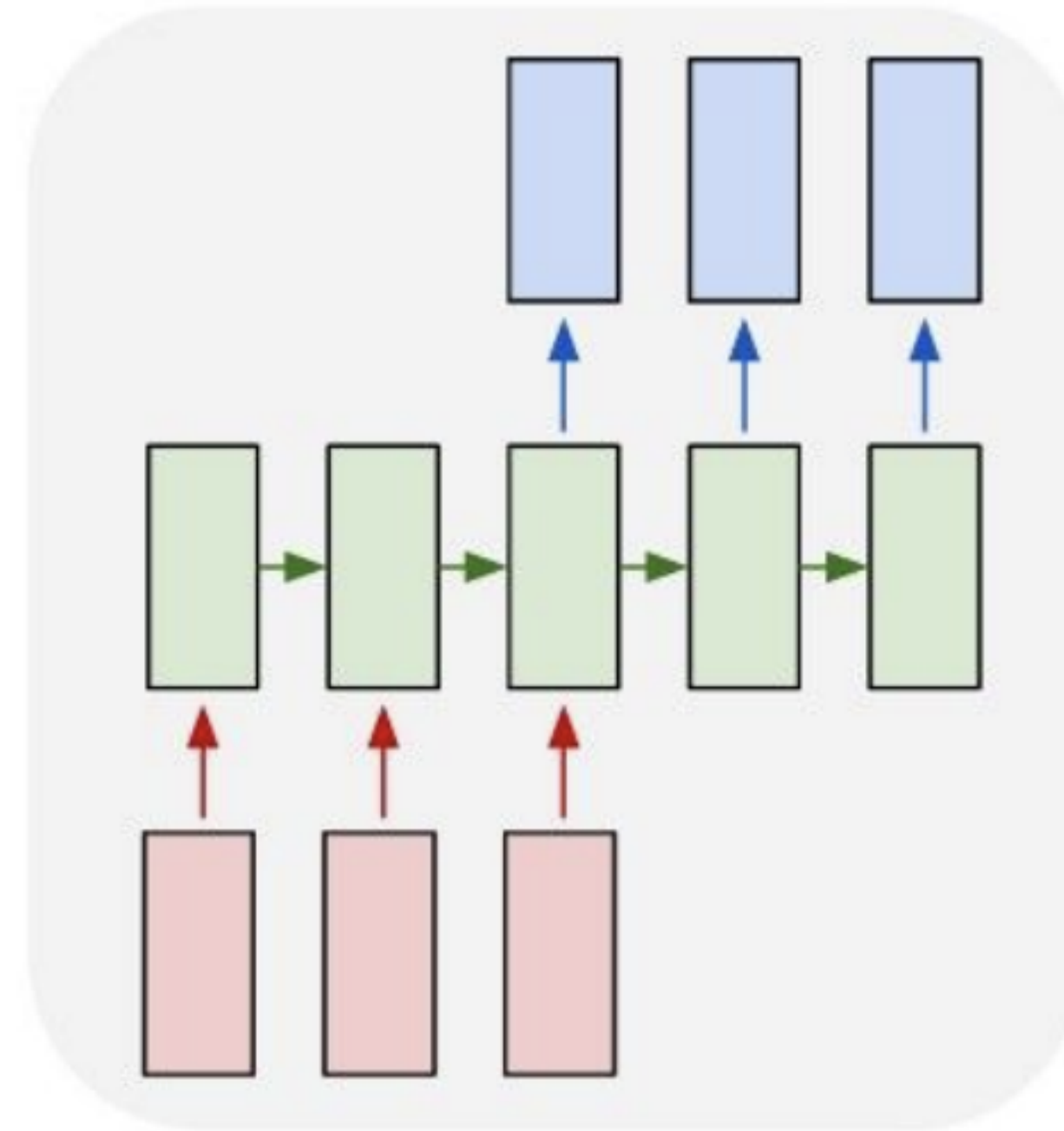
one to many



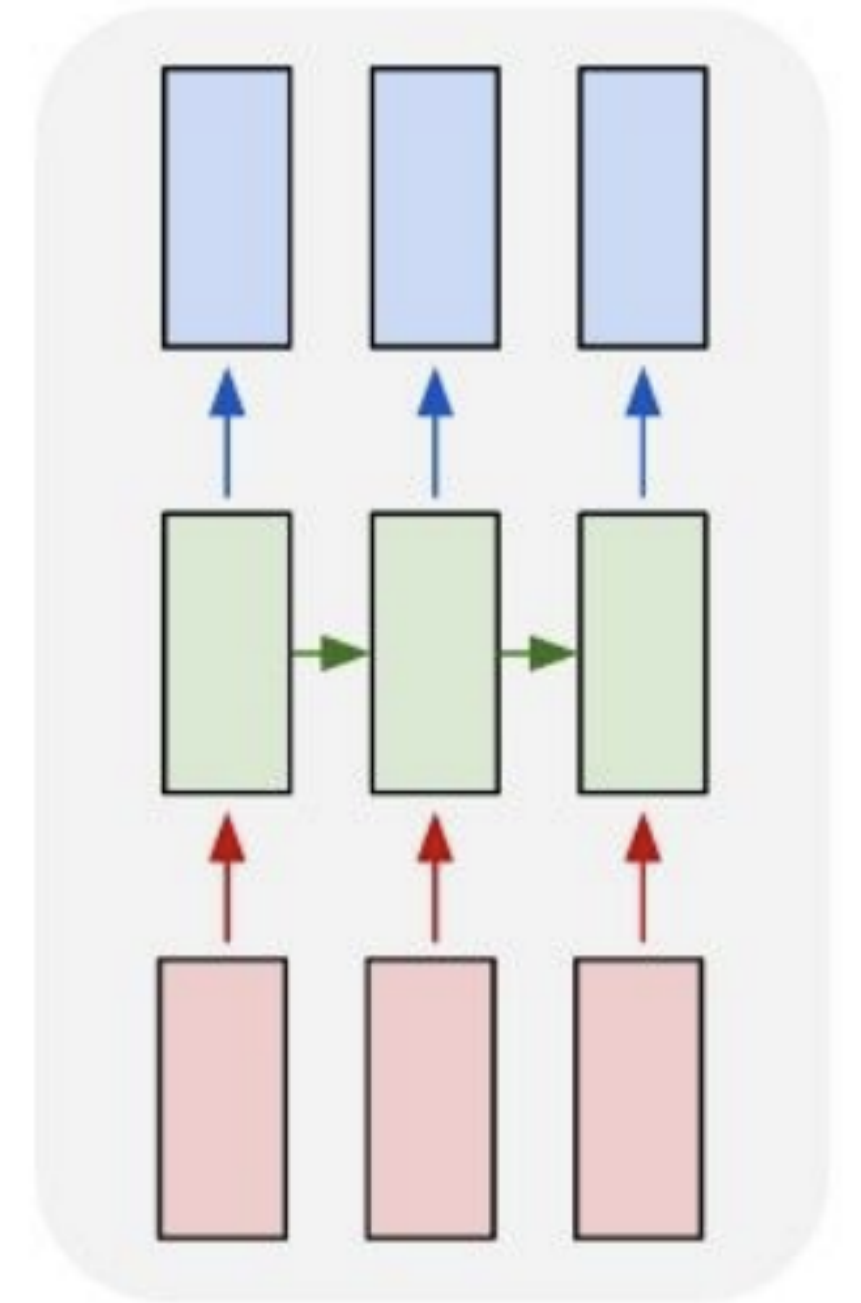
many to one



many to many



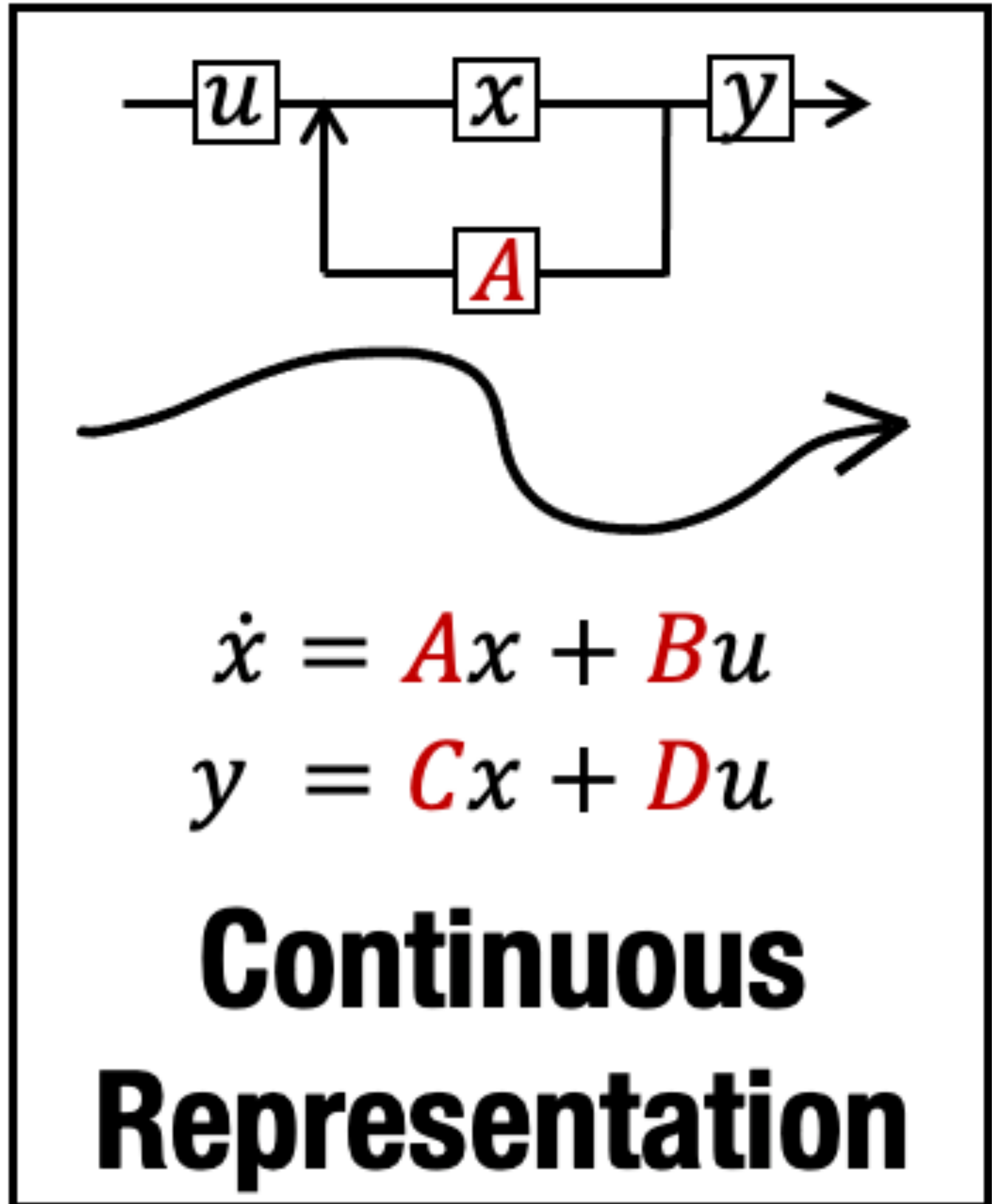
many to many



RNNs

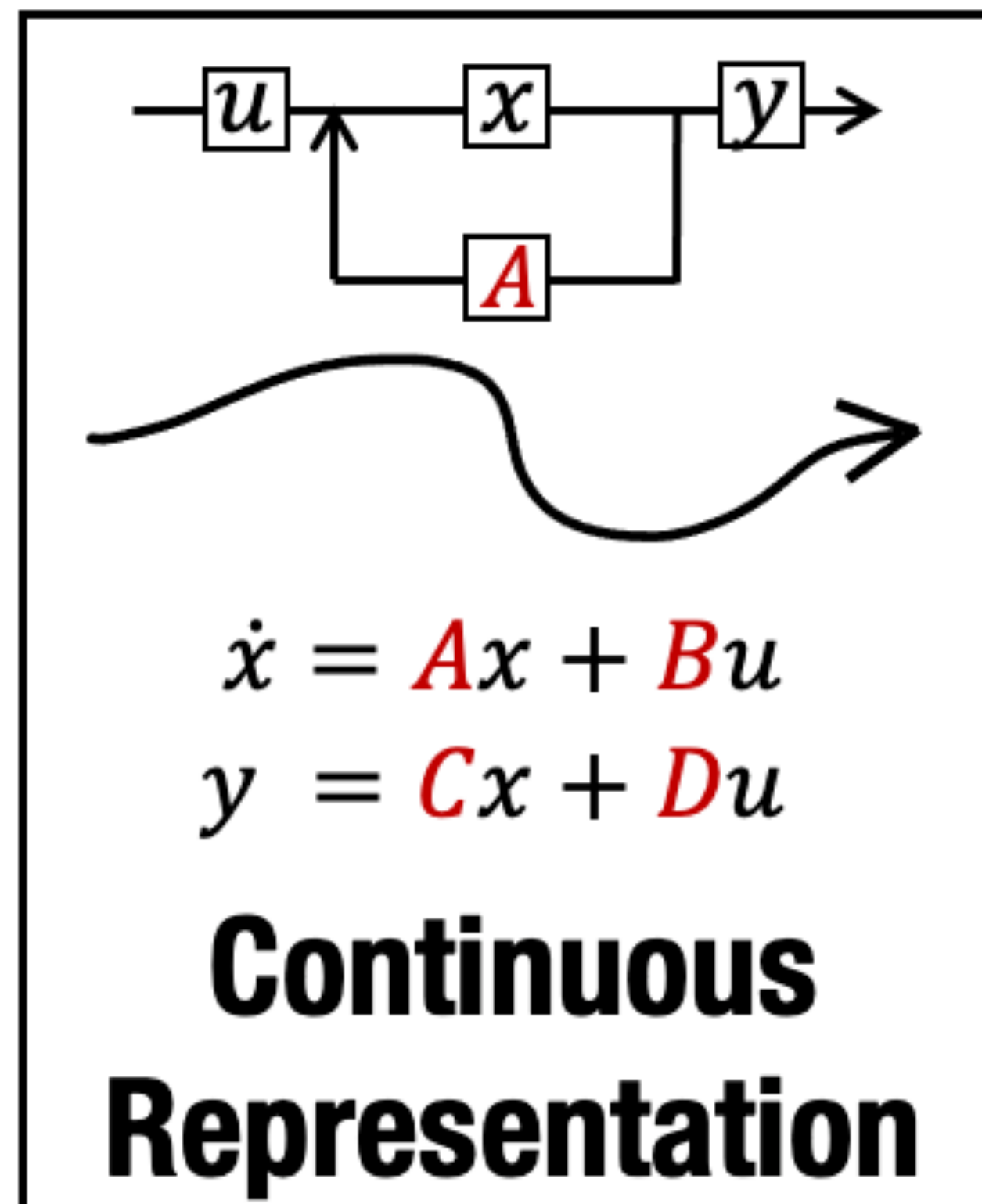
State-space models

- **Idea.** Handle sequential input using a **state-space model** (SSM)
- **Recall: Continuous SSMs**
 - Handles a sequential input $u(t)$ by accumulating the **internal state** $x(t)$
 - Sequential output $y(t)$ is determined by both $u(t), x(t)$
 - Parameterized by transition matrices A, B, C, D

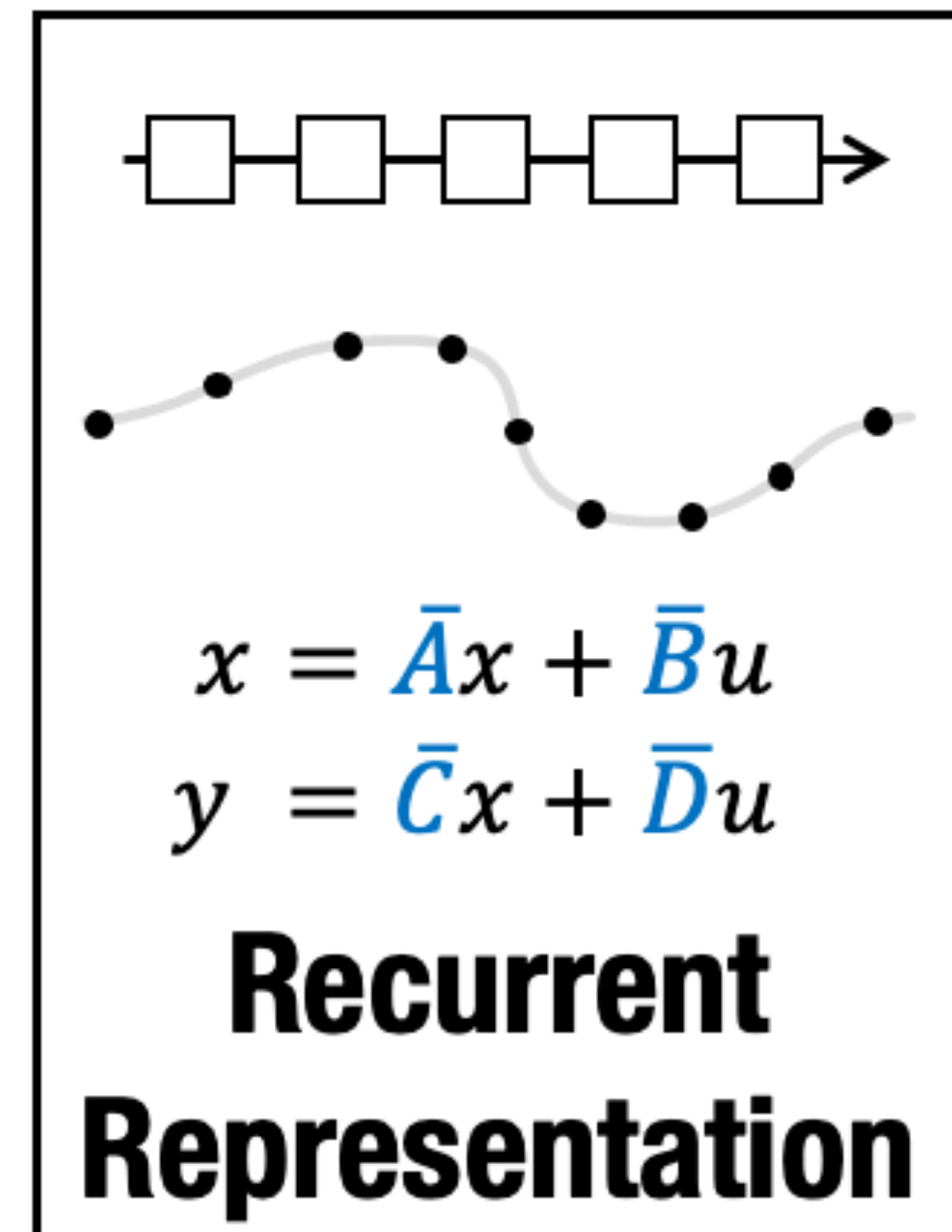


State-space models

- Discrete inputs can be handled similarly, with **discrete SSMs**
 - One can use
$$x(t) = \bar{A}x(t-1) + \bar{B}u(t)$$
$$y(t) = \bar{C}x(t) + \bar{D}u(t)$$



Discretize
→

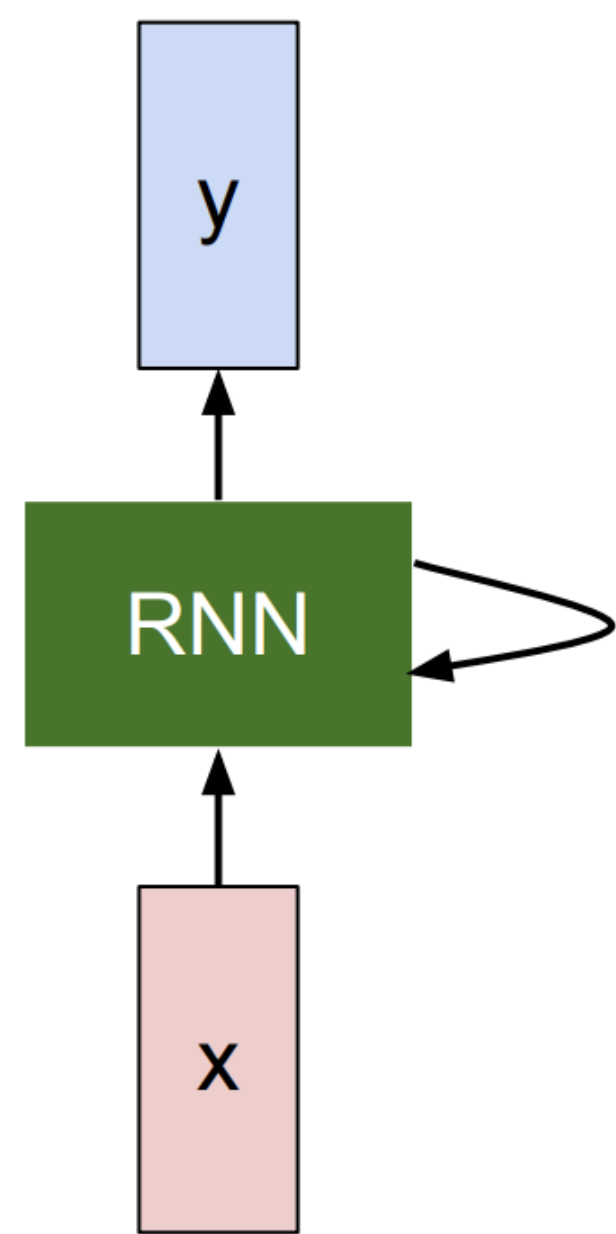


Recurrent layer

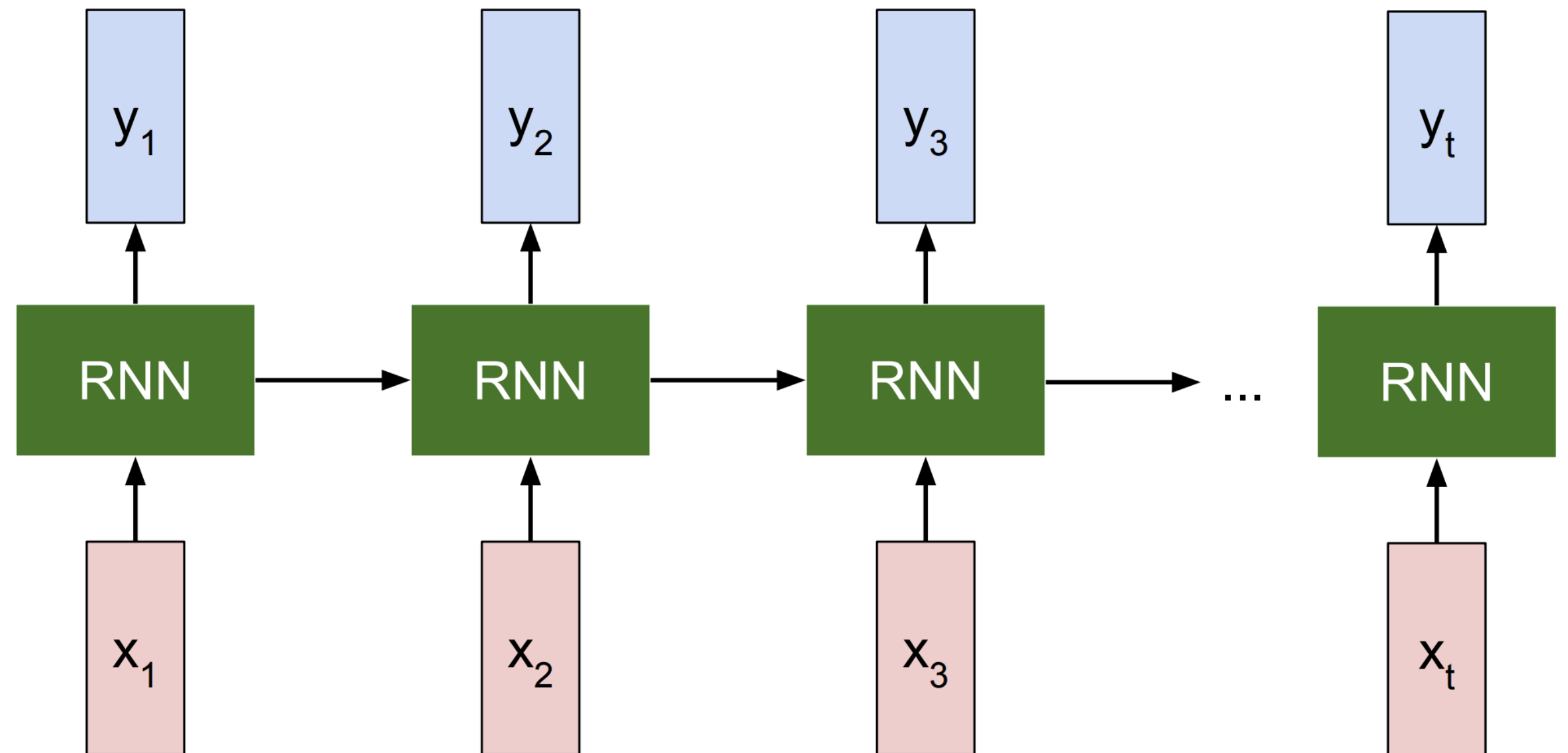
- More generally, we can use **nonlinear** modules:

- $\mathbf{h}_t = g_\theta(\mathbf{x}_t; \mathbf{h}_{t-1})$
- $\hat{\mathbf{y}}_t = f_\theta(\mathbf{x}_t; \mathbf{h}_{t-1})$

note: change of notations



RNN



RNN (unrolled)

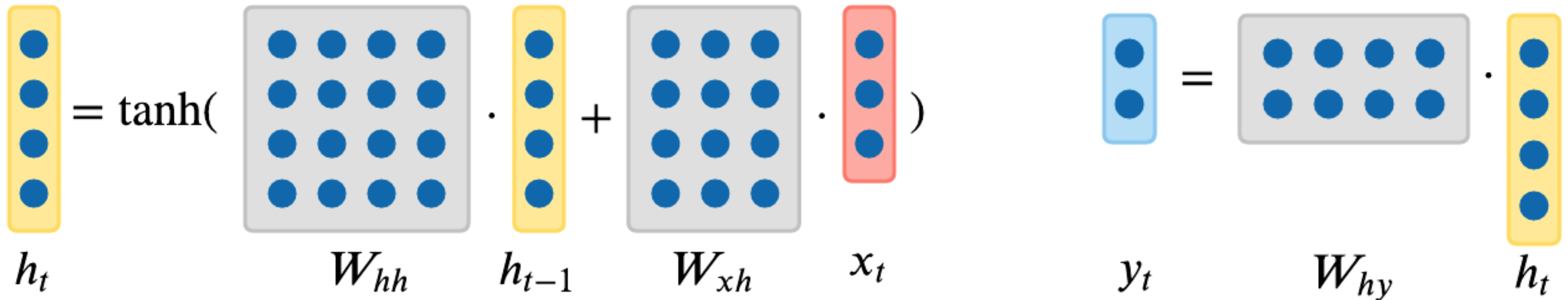
Recurrent layer

- In the simplest parameterization, the recurrence is formalized as:

$$\mathbf{h}_t = \tanh(\mathbf{W}_{hh}\mathbf{h}_{t-1} + \mathbf{W}_{xh}\mathbf{x}_t)$$

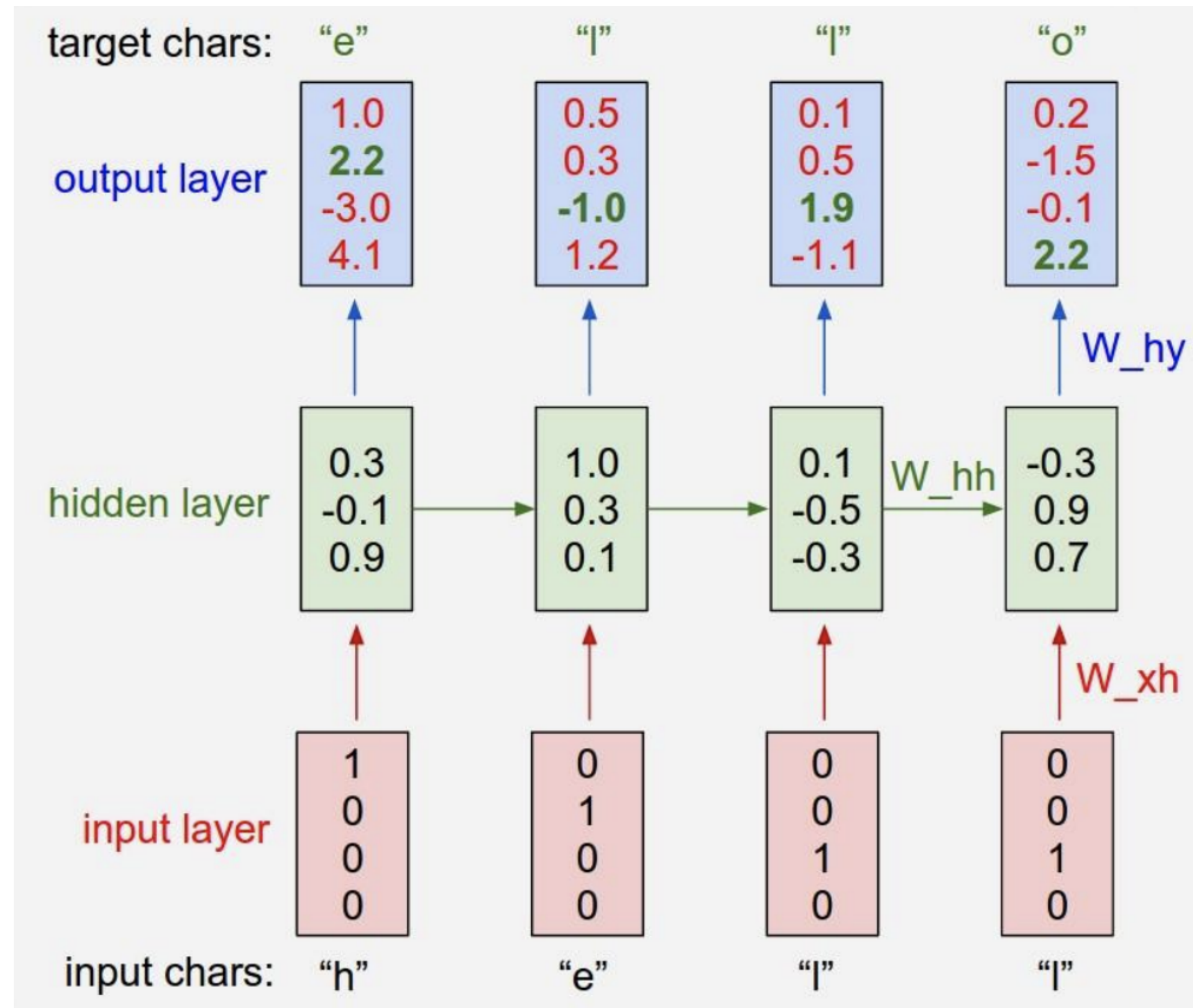
$$\mathbf{y}_t = \mathbf{W}_{hy}\mathbf{h}_t$$

Rumelhart (1986)



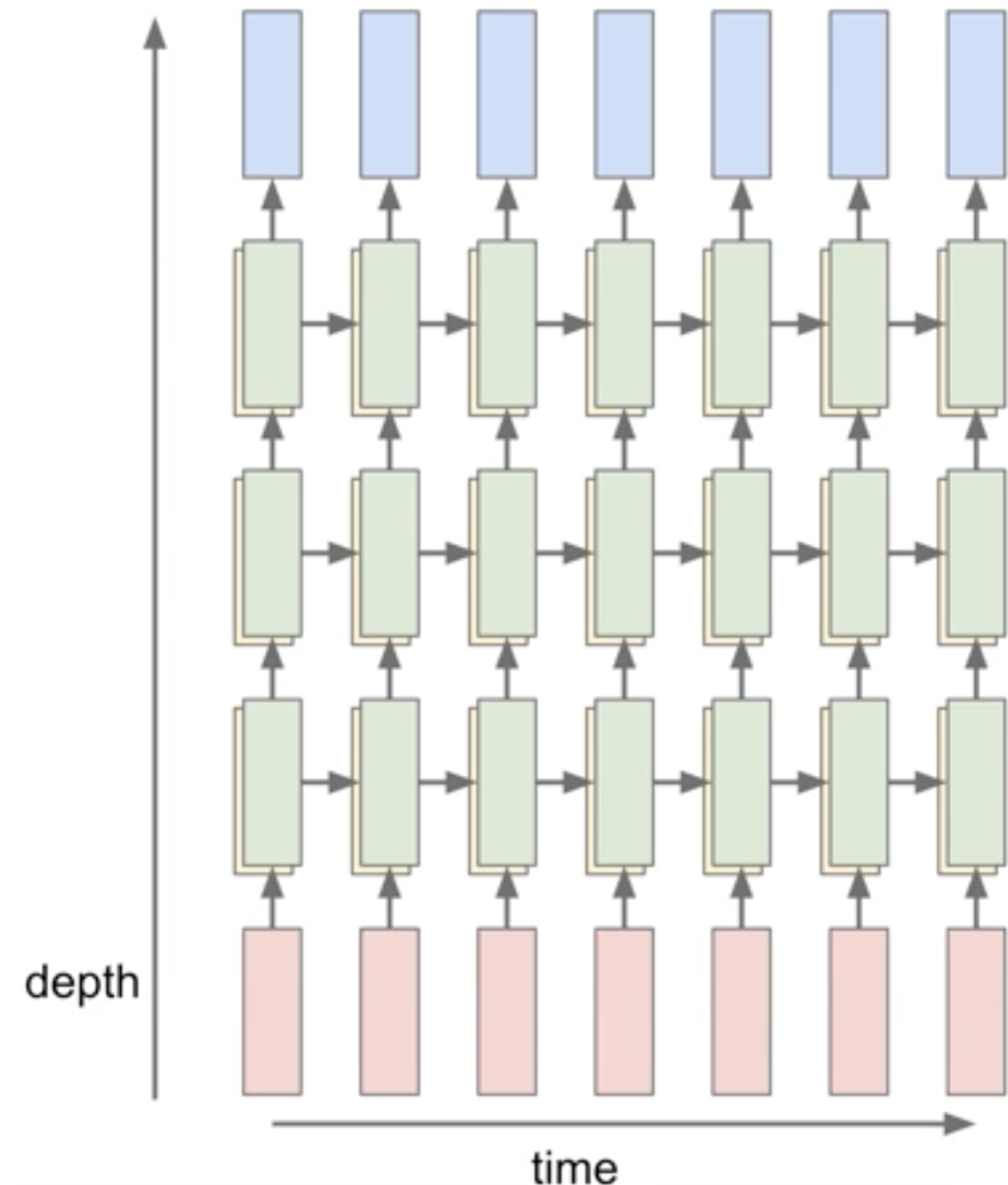
Example: RNN for sentence generation

- Consider the case with:
 - Character-level tokens
 - Single-layer RNN
 - Identity embedding
- Sentence generation can be done by feeding the **generated character as a new input**
 - Similar in GPT



Deep recurrent neural network

- Recurrent neural networks can be stacked to build a deep network
 - Strengthens the “memory”
- Quite difficult to train
 - **Vanishing/exploding gradient**



Gradients of RNNs

- Suppose that we have computed the loss at time t # i.e., L_t
 - We want to use this to update the hidden state at time 1 # i.e., $\mathbf{h}_1$

- The partial derivative of the current state w.r.t. past state is:

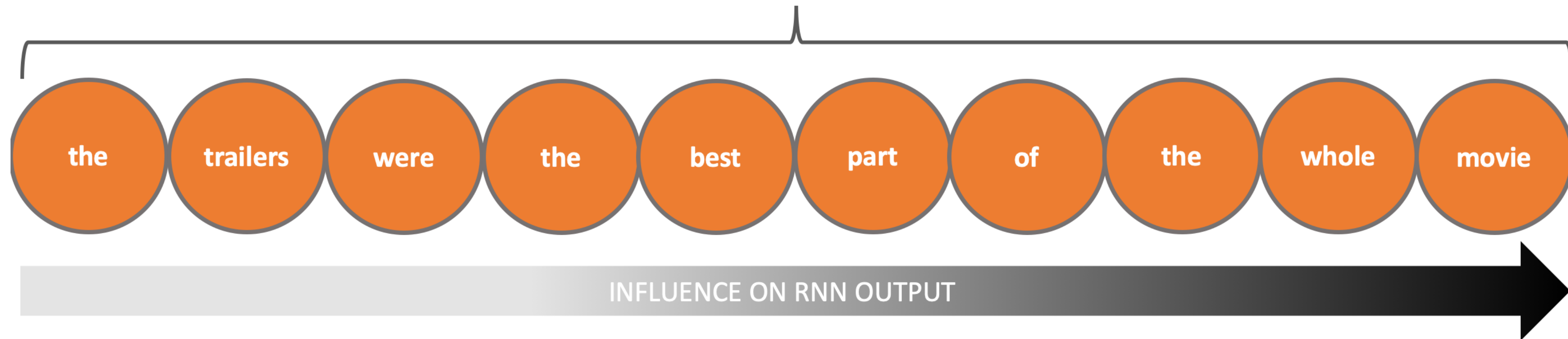
$$\frac{\partial \mathbf{h}_t}{\partial \mathbf{h}_{t-1}} = \tanh'(\mathbf{W}_{hh}\mathbf{h}_{t-1} + \mathbf{W}_{xh}\mathbf{x}_t)\mathbf{W}_{hh}$$

- Then, we have:

$$\frac{\partial L_t}{\partial \mathbf{h}_1} = \frac{\partial L_t}{\partial \mathbf{h}_t} \cdot \frac{\partial \mathbf{h}_t}{\partial \mathbf{h}_{t-1}} \cdot \dots \cdot \frac{\partial \mathbf{h}_2}{\partial \mathbf{h}_1} = \frac{\partial L_t}{\partial \mathbf{h}_t} \cdot \left(\prod_{i=2}^t \tanh'(\mathbf{W}_{hh}\mathbf{h}_{i-1} + \mathbf{W}_{xh}\mathbf{x}_i) \right) \mathbf{W}_{hh}^{t-1}$$

Gradients of RNNs

“the trailers were the best part of the whole movie.”



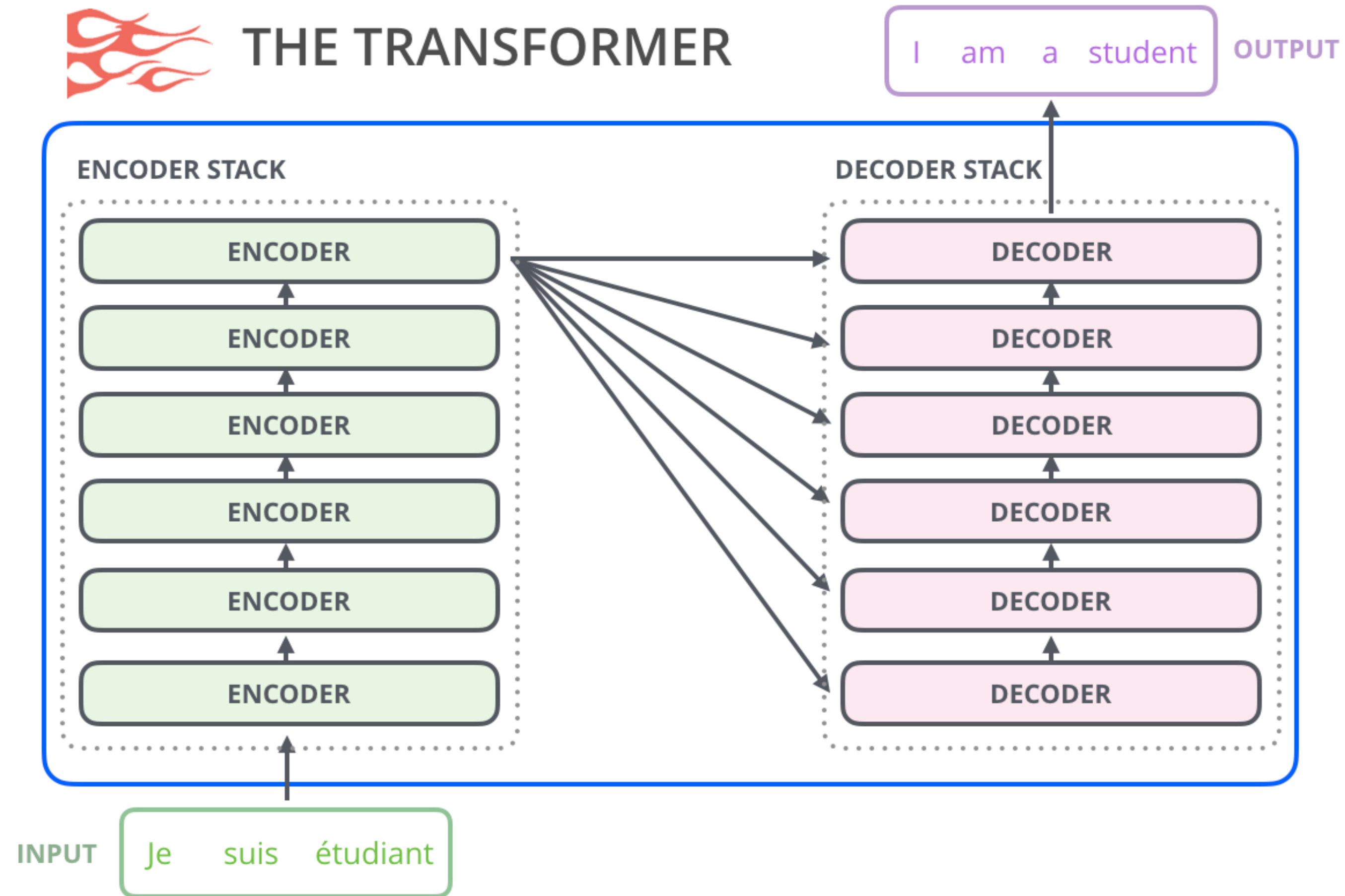
- **Solution.**

- Adopt **extra modules** that are designed for long-term dependencies
 - e.g., LSTM
- Let the very old input **directly affect** the new output
 - e.g., Transformers

Transformers

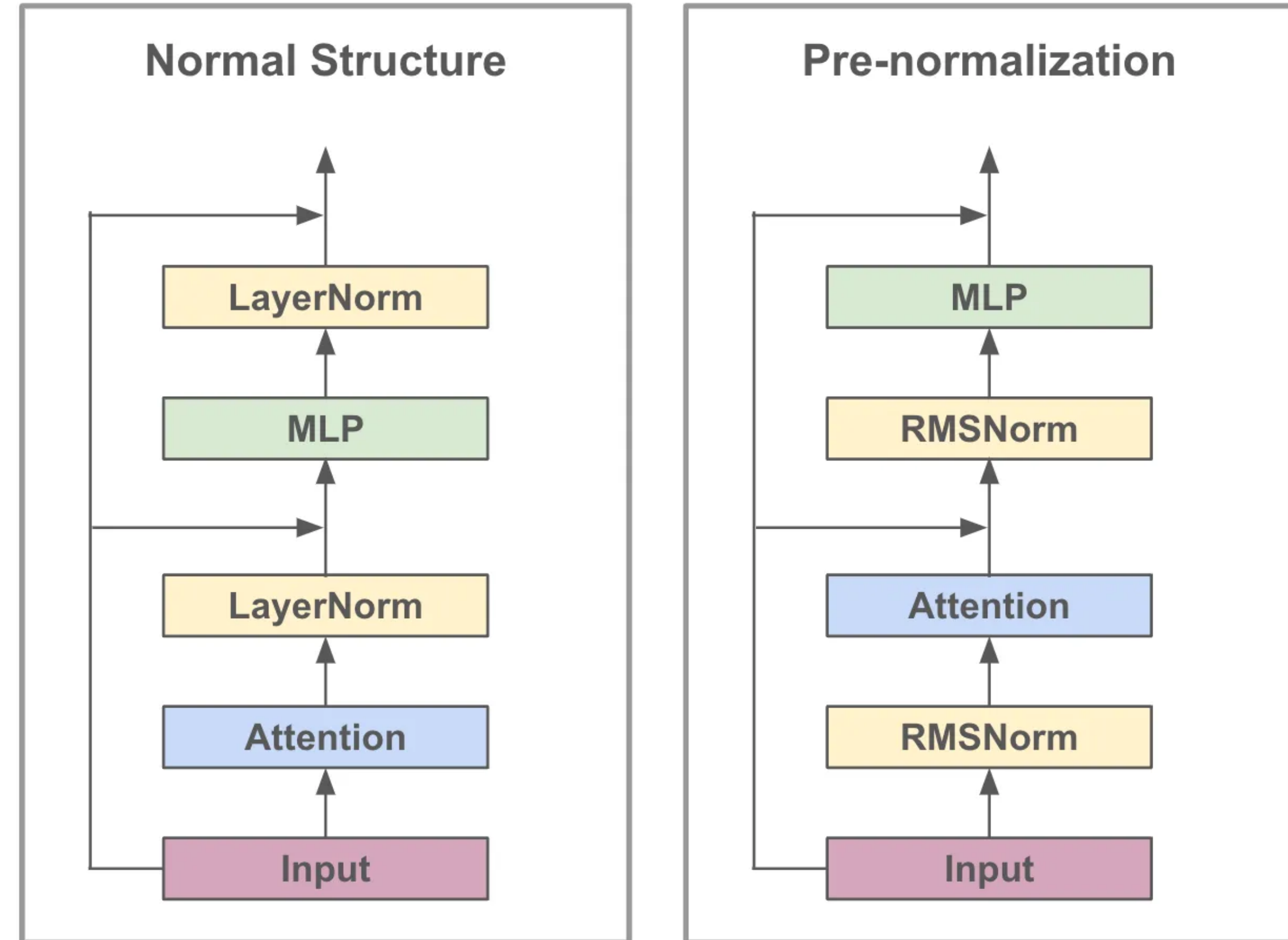
Transformers

- Consists of:
 - a stack of encoder blocks
 - a stack of decoder blocks
- **Encoder-only.** BERT
- **Decoder-only.** GPT (our focus)



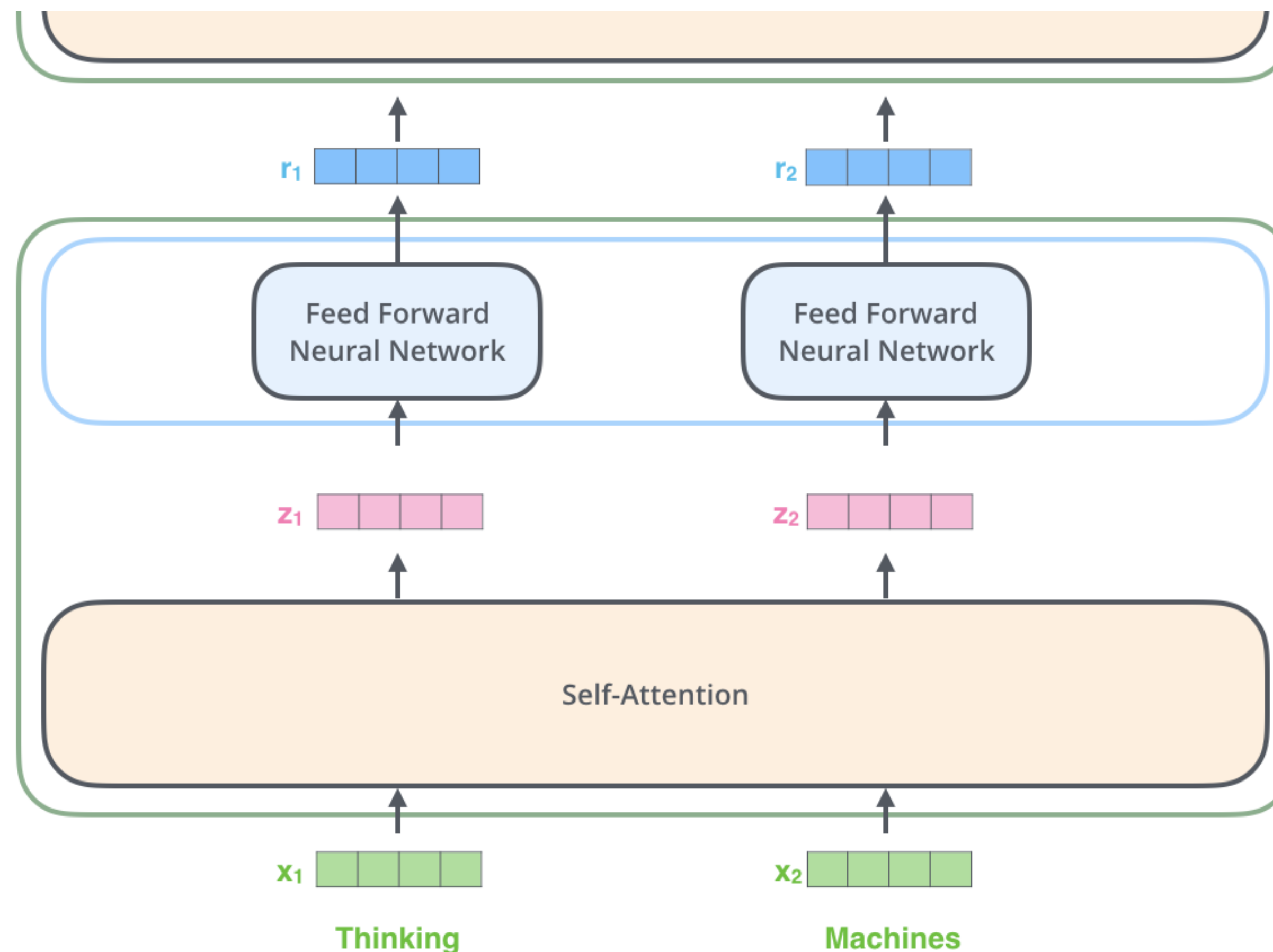
Transformers

- Each block consists of four elements
 - Multi-head Self Attention (MHA)
 - Feedforward network (FFN)
 - LayerNorm / RMSNorm
 - Residual connection
- Only new component is the MHA



MHA and FFN

- MHA and FFN plays a complementary role
 - **MHA.** Models **inter-token** interaction
 - **FFN.** Applies **intra-token** operation # i.e., same op. for all tokens



Self-Attention

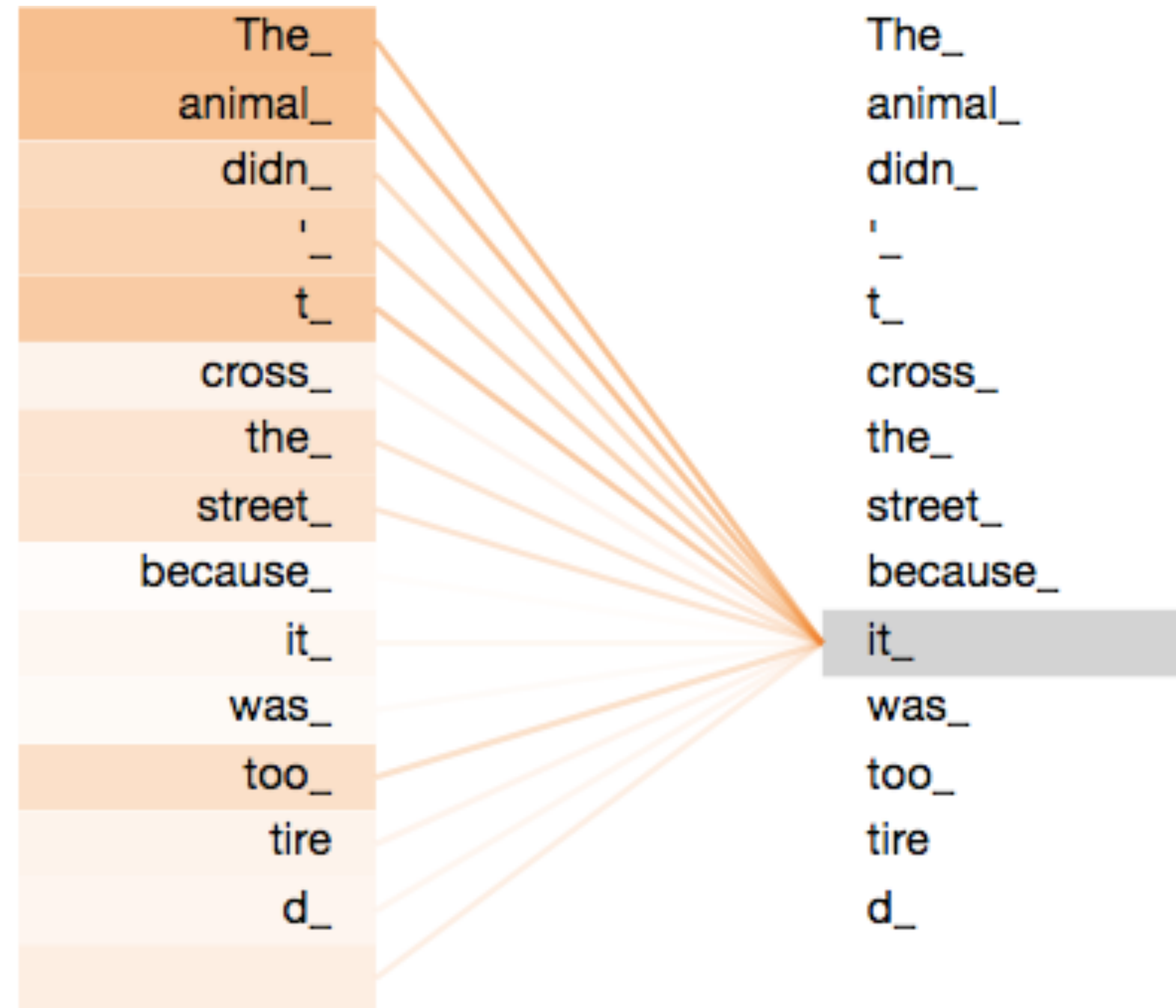
- Roughly, the self-attention layer does two things

(1) For i-th token, we measure the **relevance** of {1,...,N}th tokens

- called “attention score”

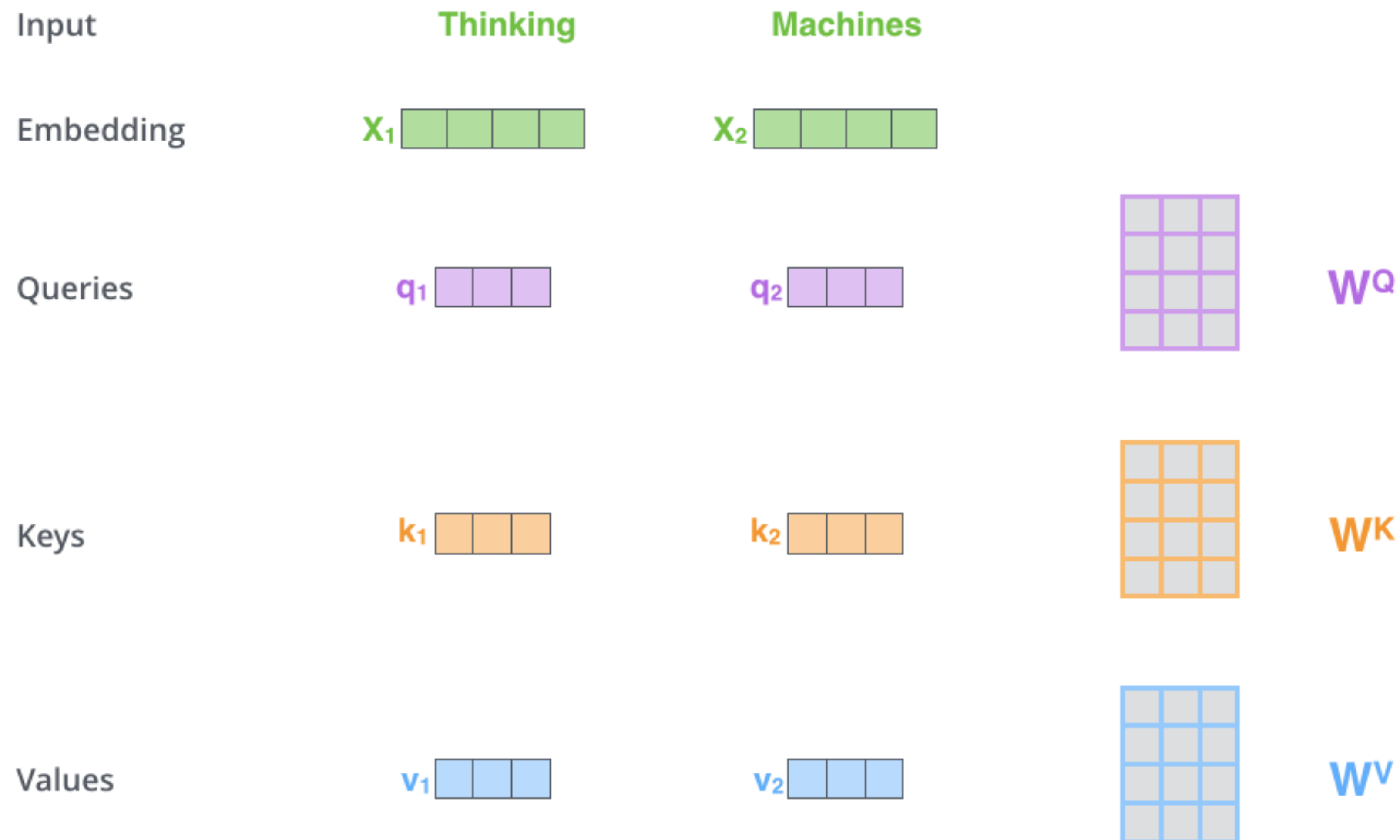
(2) Take a relevance-weighted sum of other token’s “values” to compute the i-th token’s output

$$o_i = \sum_j \text{attention}(i, j) \cdot \text{val}(j)$$



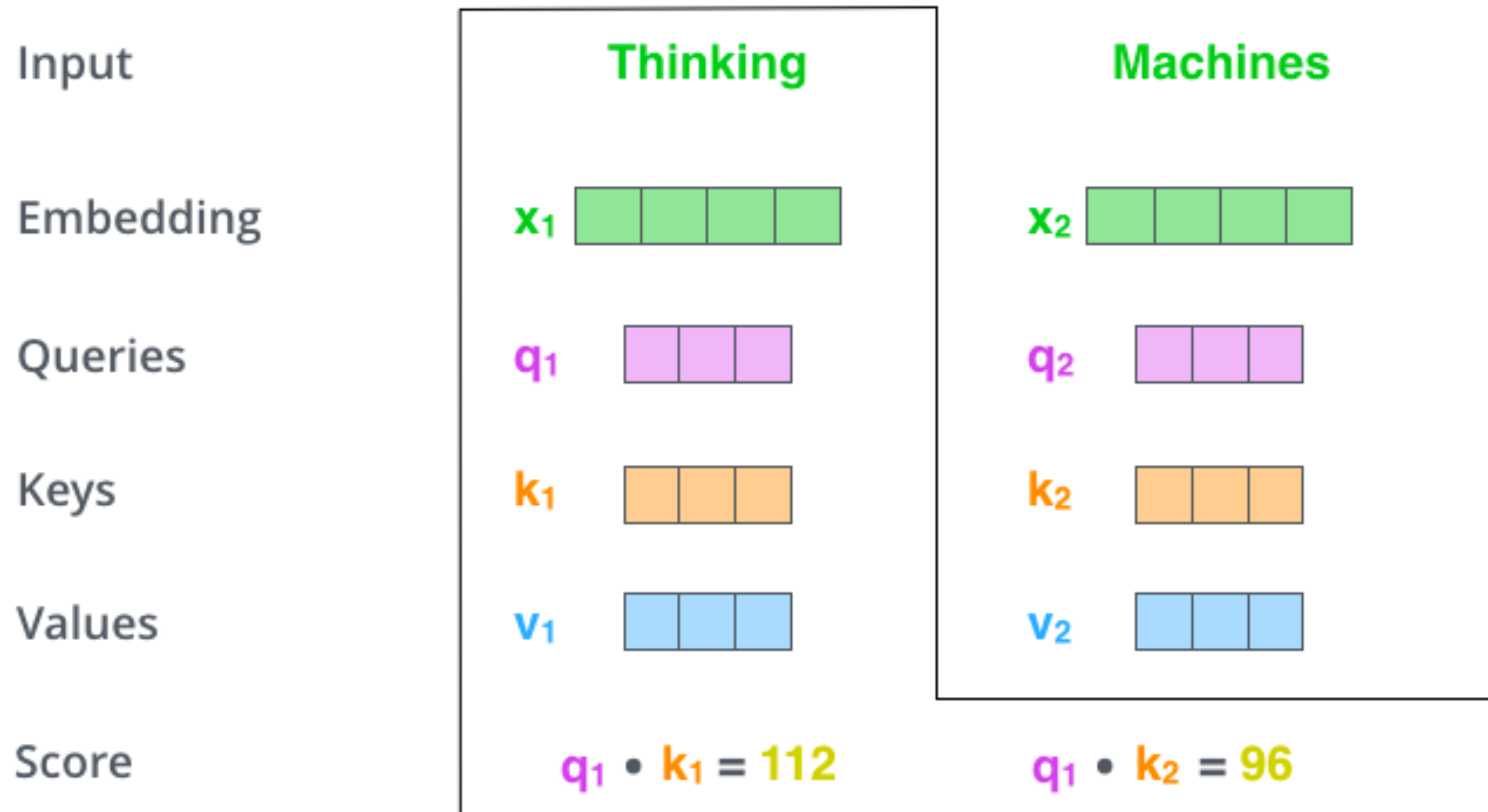
Self-Attention

- **Step 1.** For each **token**, we compute **query**, **key**, and **value**
 - Weight matrices are shared over all tokens



Self-Attention

- **Step 2.** Compute **dot products** of the **query** (self) and **key** (self, others)



Self-Attention

- **Step 3.** Compute **output** as a weighted sum of **values**
 - weighted by the **softmax of dot products** (attention score)
 - normalized by the dimensions

The diagram illustrates the calculation of the self-attention output. It shows a purple 2x3 matrix labeled **Q** multiplied by an orange 3x2 matrix labeled **K^T**. The result of this multiplication is divided by $\sqrt{d_k}$ and then passed through a **softmax** function, all enclosed in a yellow rounded rectangle. To the right of this rectangle is a blue 2x3 matrix labeled **V**. Below the yellow rectangle is a pink 2x3 matrix labeled **Z**, preceded by an equals sign, indicating that **Z** is the result of the softmax operation.

$$\text{softmax} \left(\frac{\begin{matrix} \text{Q} \\ \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} \end{matrix} \times \begin{matrix} \text{K}^T \\ \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \end{matrix}}{\sqrt{d_k}} \right) \begin{matrix} \text{V} \\ \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} \end{matrix}$$
$$= \begin{matrix} \text{Z} \\ \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} \end{matrix}$$

Computation & Memory

- Suppose that we have n tokens
 - Q/K/V computation: $O(n)$
 - Attention: $O(n^2)$
 - Weighted sum: $O(n^2)$
- Unlike RNNs, requires **quadratic** operations with respect to the sequence length!
 - Why we need many GPUs

Input

Embedding

Queries

Keys

Values

Score

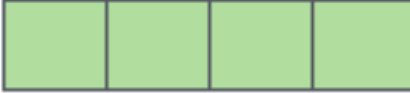
Divide by 8 ($\sqrt{d_k}$)

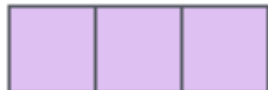
Softmax

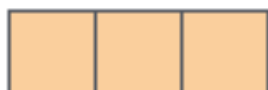
Softmax
X
Value

Sum

Thinking

x_1 

q_1 

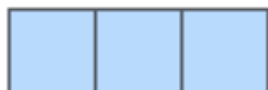
k_1 

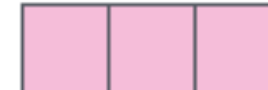
v_1 

$q_1 \cdot k_1 = 112$

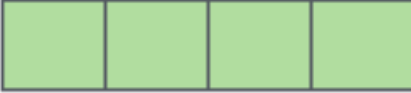
14

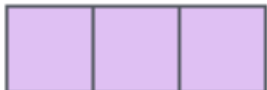
0.88

v_1 

z_1 

Machines

x_2 

q_2 

k_2 

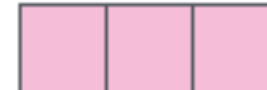
v_2 

$q_1 \cdot k_2 = 96$

12

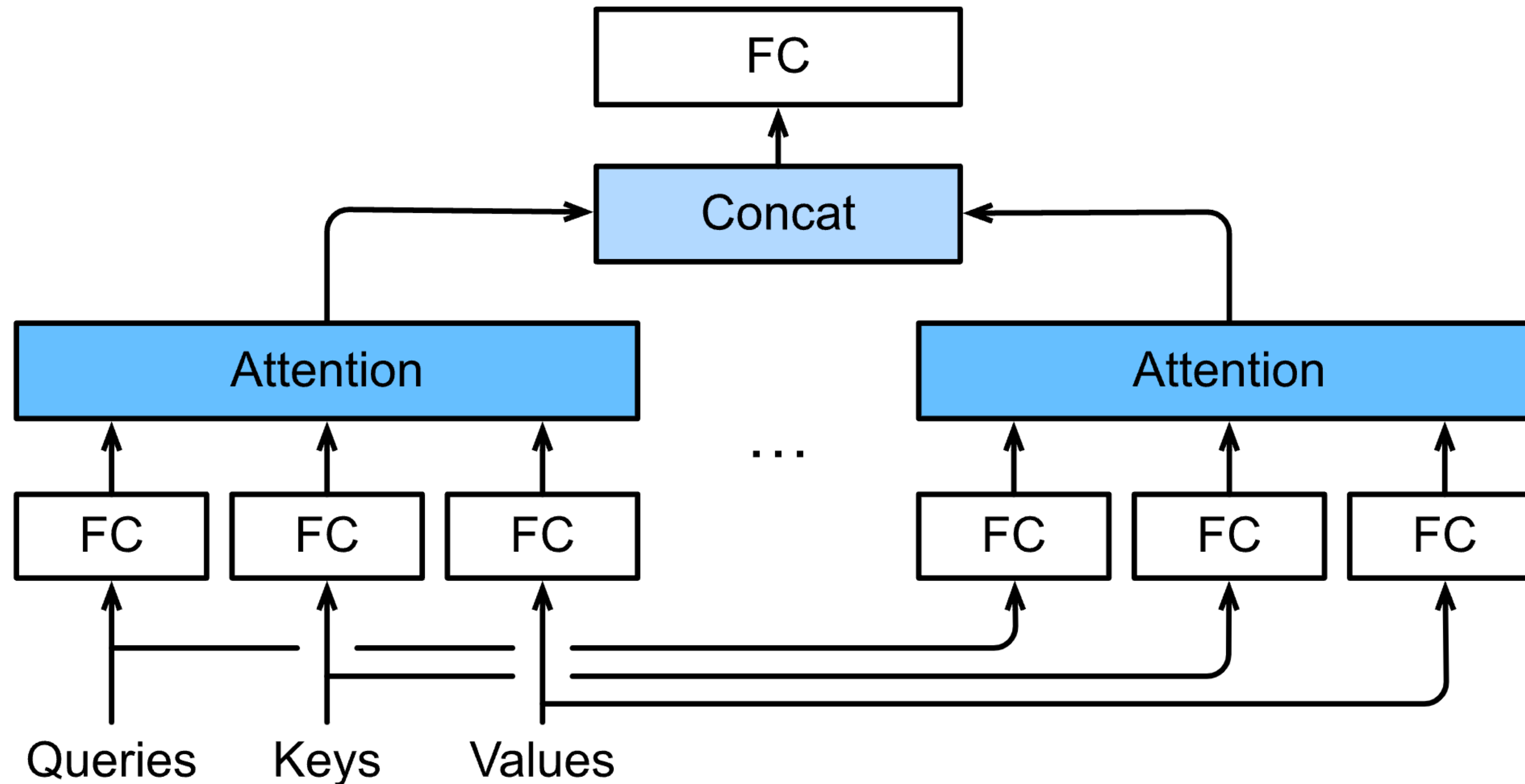
0.12

v_2 

z_2 

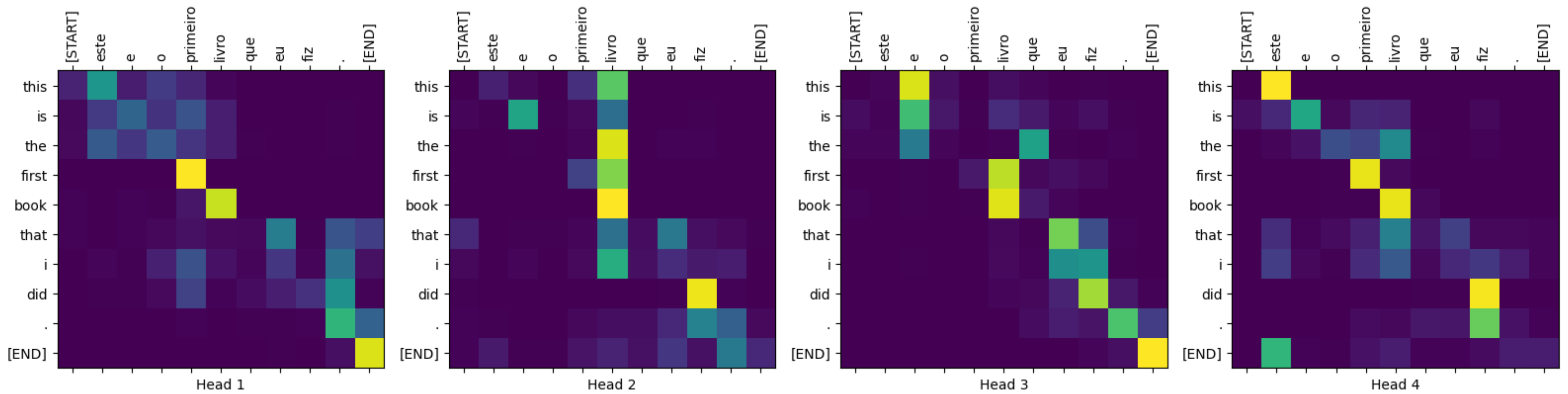
Multi-head Self-Attention

- Typically, we use **multiple self-attention layers** in a transformer block
 - Computed in parallel
 - Outputs are concatenated and linearly projected



Multi-head Self-Attention

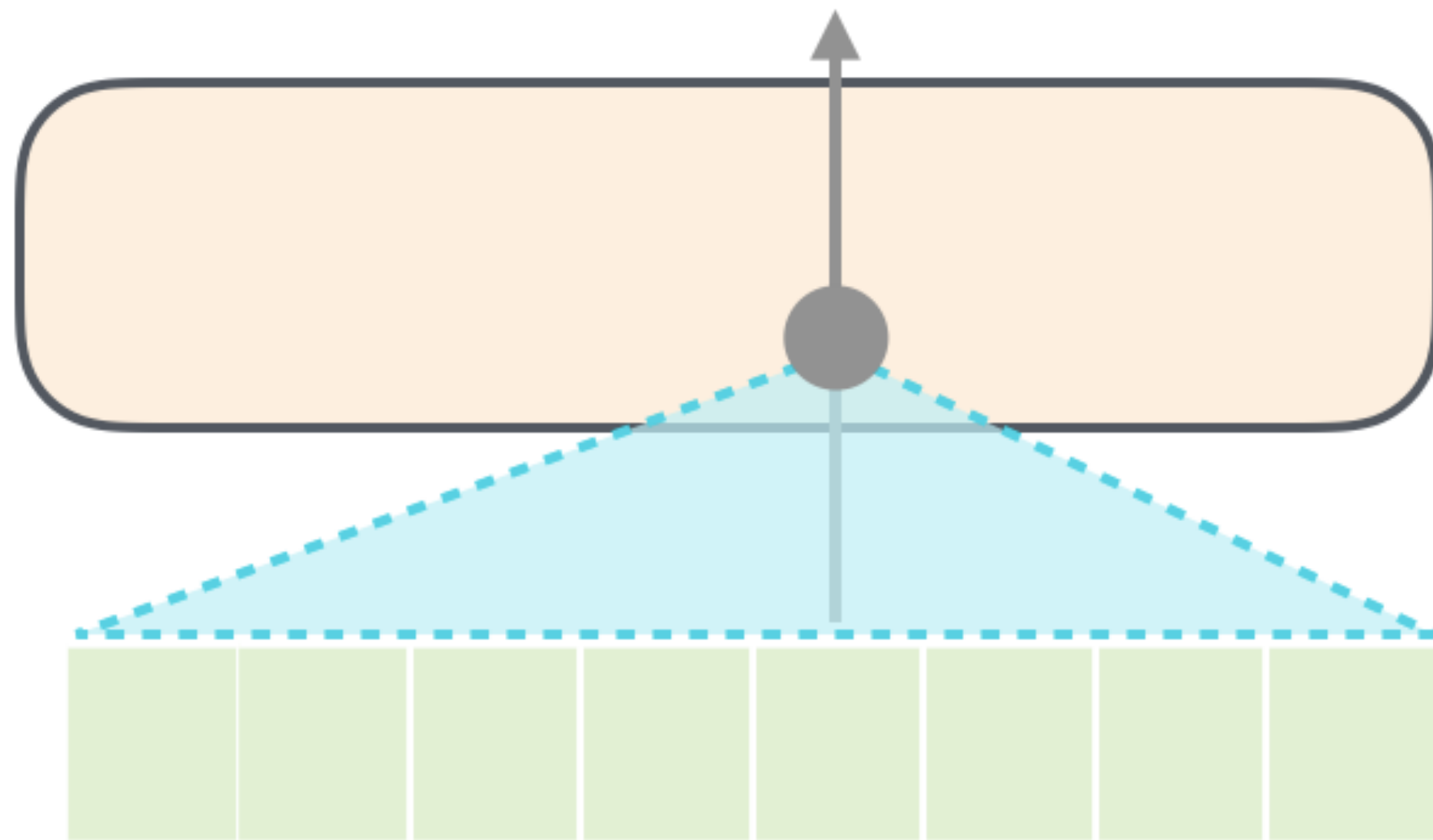
- Each attention head tend to capture diverse attention patterns
 - Similar to multiple convolution filters in a layer



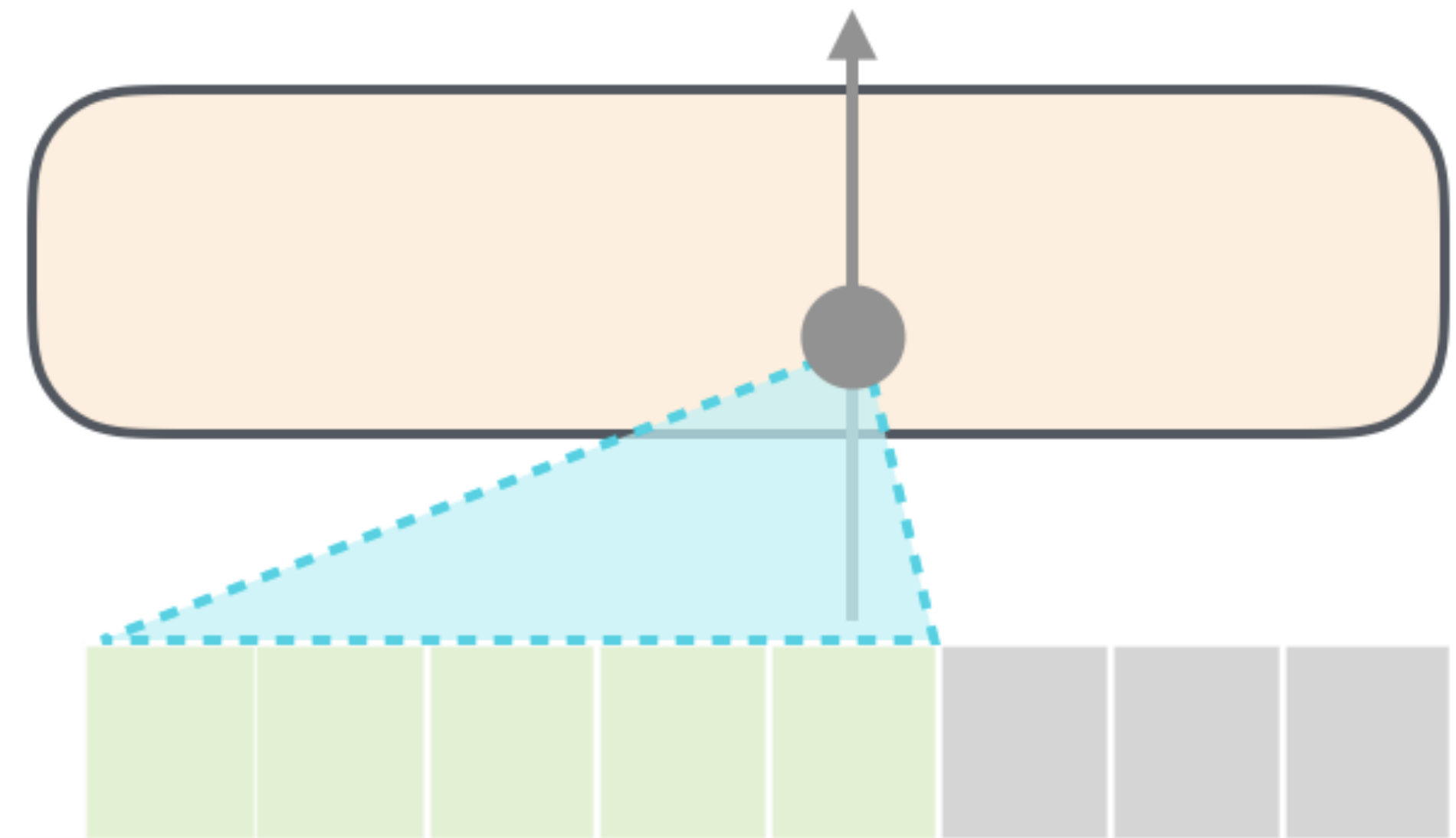
Causal masking for attention

- In decoder-only models (like GPT), the self-attention layers are **masked**
 - For generating t -th token, the model can only utilize $\mathbf{x}_1, \dots, \mathbf{x}_{t-1}$

Self-Attention



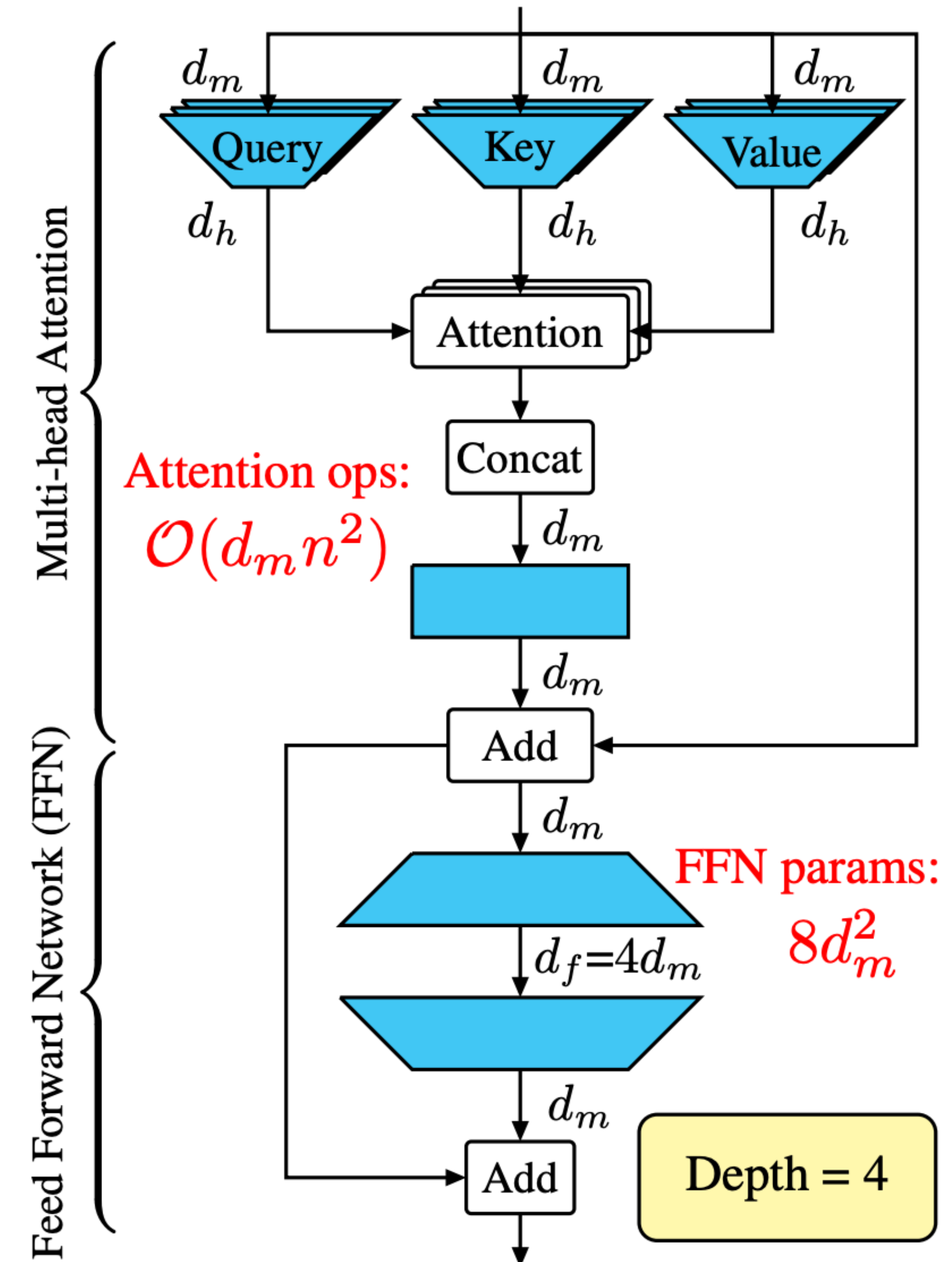
Masked Self-Attention



Feedforward network

- Fully-connected layers that follow the MHA
 - Basic.** Use two-layer MLP
 - Inverted bottleneck structure
- Tend to be very compute-heavy
 - Especially so for larger models

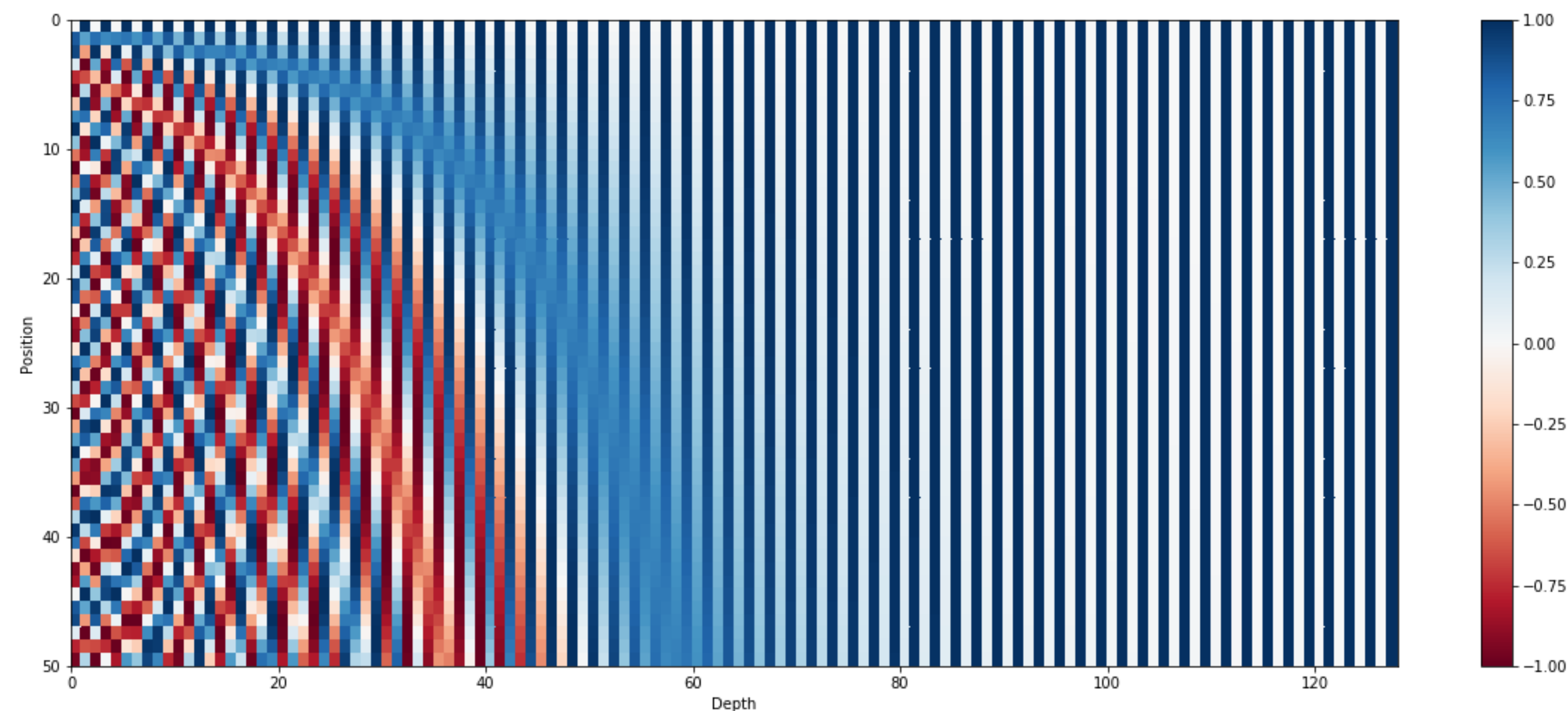
1	description	FLOPs / update	% FLOPS MHA	% FLOPS FFN	% FLOPS attn	% FLOPS logit
8	OPT setups					
9	760M	4.3E+15	35%	44%	14.8%	5.8%
10	1.3B	1.3E+16	32%	51%	12.7%	5.0%
11	2.7B	2.5E+16	29%	56%	11.2%	3.3%
12	6.7B	1.1E+17	24%	65%	8.1%	2.4%
13	13B	4.1E+17	22%	69%	6.9%	1.6%
14	30B	9.0E+17	20%	74%	5.3%	1.0%
15	66B	9.5E+17	18%	77%	4.3%	0.6%
16	175B	2.4E+18	17%	80%	3.3%	0.3%



Positional encoding

- **Problem.** Self-attention ignores the positional information of each token
- Solution. Add **position-specific vector** to the token embedding
 - called positional encoding
 - added at initial embedding or in each block

$$\vec{p}_t^{(i)} = f(t)^{(i)} := \begin{cases} \sin(\omega_k \cdot t), & \text{if } i = 2k \\ \cos(\omega_k \cdot t), & \text{if } i = 2k + 1 \end{cases} \quad \omega_k = \frac{1}{10000^{2k/d}}$$



More references

- **Beginner.** Jay Alammam's blog posts
 - <https://jalammar.github.io/illustrated-transformer/>
- **Advanced.**
 - Phuong and Hutter, "Formal algorithms for Transformers," 2022
 - <https://arxiv.org/abs/2207.09238>
 - He and Hoffman, "Simplifying Transformer Blocks," 2023
 - <https://arxiv.org/abs/2311.01906>

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